

Multiple Choice

1.(5 pts.) Find the foci of the ellipse $\frac{x^2}{4} + \frac{y^2}{13} = 1$.

(a) $(3, 0), (-3, 0)$ (b) $(0, \sqrt{17}), (0, -\sqrt{17})$ (c) $(3, \sqrt{17}), (-3, \sqrt{17})$

(d) $(\sqrt{17}, 0), (-\sqrt{17}, 0)$ (e) $(0, 3), (0, -3)$

2.(5 pts.) Find the equation of the hyperbola having foci $(6, 0), (-6, 0)$ and eccentricity $e = \frac{6}{5}$.

(a) $\frac{x^2}{11} - \frac{y^2}{25} = 1$ (b) $\frac{x^2}{25} - \frac{y^2}{36} = 1$ (c) $\frac{x^2}{36} - \frac{y^2}{25} = 1$

(d) $\frac{x^2}{25} - \frac{y^2}{11} = 1$ (e) $\frac{x^2}{11} - \frac{y^2}{36} = 1$

3.(5 pts.) Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{3^{n^2}}$$

The radius of convergence is 3.

(a) $(2, 8)$ (b) $(2, 8]$ (c) $(-\infty, \infty)$ (d) $[2, 8)$ (e) $[2, 8]$

4.(5 pts.) Find the first 4 nonzero terms of the Maclaurin series for

$$\frac{\cos x}{(1-x)^2}$$

Hint: Multiply a power series for $\cos x$ by a power series for $\frac{1}{1-x}$.

(a) $x + \frac{1}{2!}x^3 + \frac{1}{4!}x^5 - \frac{1}{6!}x^6$

(b) $1 + x + x^2 + x^3$

(c) $1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3$

(d) $1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6$

(e) $1 - x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3$

5.(5 pts.) Find the third Taylor polynomial $P_3(x)$ for $f(x) = \frac{1}{x}$ at $a = 2$.

(a) $1 - x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3$

(b) $(x-2) + \frac{1}{2!}(x-2)^3 + \frac{1}{4!}(x-2)^5 - \frac{1}{6!}(x-2)^6$

(c) $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{3}{48}(x-2)^3$ (d) $\frac{1}{2} + \frac{1}{4}(x-2) + \frac{1}{4}(x-2)^2 + \frac{3}{8}(x-2)^3$

(e) $1 + x + x^2 + x^3$

6.(5 pts.) Which one of the following series converges conditionally?

- (a) $\sum_{n=1}^{\infty} n$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{\frac{1}{3}} + 1}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$
(d) $\sum_{n=1}^{\infty} \frac{1}{n^5}$ (e) $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{3}}}$

7.(5 pts.) Find the first three terms of the binomial series

$$\frac{1}{(1+x)^{\frac{1}{2}}} = (1+x)^{-\frac{1}{2}}$$

- (a) $1 - \frac{x}{2} + \frac{3x^2}{8}$ (b) $1 - x + x^2$ (c) $1 + x + x^2$
(d) $1 + \frac{x}{2} + \frac{3x^2}{8}$ (e) $1 - \frac{x^2}{2} + \frac{x^4}{4!}$

8.(5 pts.) Which of the following series converge?

- (1) $\sum_{n=0}^{\infty} \frac{n}{(n^7 + 1)^{\frac{1}{3}}}$
(2) $\sum_{n=0}^{\infty} \frac{n2^{3n}}{3^{2n}}$
(3) $\sum_{n=0}^{\infty} \frac{n!}{2^n}$

- (a) (1) and (2) (b) (1) only (c) (2) only
(d) none of the above (e) (1), (2) and (3)

9.(5 pts.) Find Taylor's remainder $R_3(2)$ for the function $f(x) = e^{2x}$ at $a = 0$.

- (a) $\frac{16e^c}{4!}(2)^4$ for some $0 < c < 2$ (b) $\frac{16e^2}{4!}(c)^4$ for some $0 < c < 2$
(c) $\frac{8e^c}{3!}(2)^3$ for some $0 < c < 2$ (d) $\frac{16e^c}{4!}(2)^4$ for some $2 < c$
(e) $\frac{8e^c}{3!}(2)^3$ for some $c > 2$

10.(5 pts.) Find the radius of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{12^n}$$

- (a) 24 (b) 0 (c) ∞ (d) 6 (e) 12

Partial Credit

11.(10 pts.)

a. Show that the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$$

converges by applying the Alternating series test. To receive credit you should explicitly verify the hypotheses of this test.

b. Use the Integral Test to show that this series does not absolutely converge. Again you must check the necessary hypotheses.

12.(10 pts.) Write down the first four non-zero terms of the MacLaurin series solution to the differential equation with initial value

$$y' - 2y = 0, \quad y(0) = 1.$$

13.(10 pts.) Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^{2n}}{2^n}$$

14.(10 pts.) Find the Maclaurin series for each of the following functions:

(a) $\frac{1}{1-2x}$

(b) $\frac{2}{(1-2x)^2}$ (hint: differentiate your answer to (a))

(c) $\frac{2x}{(1-2x)^2}$

15.(10 pts.) Given the function $f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{nx^{n-1}}{11^n}$, let $F(x) = \int_0^x f(t)dt$. Use

what you know about alternating series to estimate how closely the order 3 MacLaurin polynomial approximates $F(x)$ for any x in the interval $[0, 1]$.

Name: _____

Instructor: Bullwinkle

Exam III
April 25, 2000

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (●) |
| 2. | (a) | (b) | (c) | (●) | (e) |
| 3. | (a) | (b) | (c) | (●) | (e) |
| 4. | (a) | (b) | (●) | (d) | (e) |
| 5. | (a) | (b) | (●) | (d) | (e) |
| 6. | (a) | (●) | (c) | (d) | (e) |
| 7. | (●) | (b) | (c) | (d) | (e) |
| 8. | (●) | (b) | (c) | (d) | (e) |
| 9. | (●) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (●) |

DO NOT WRITE IN THIS BOX!

Total multiple choice: _____

11. _____

12. _____

13. _____

14. _____

15. _____

Total: _____