

### Multiple Choice

1.(5 pts.) The function  $x^x$  has a global minimum on the interval  $x > 0$ . (You need not check this.) This minimum value occurs at

- (a)  $x = \ln e - 1$  (b)  $x = \frac{1}{2}$  (c)  $x = \ln 2$  (d)  $x = \frac{1}{e}$  (e)  $x = e$

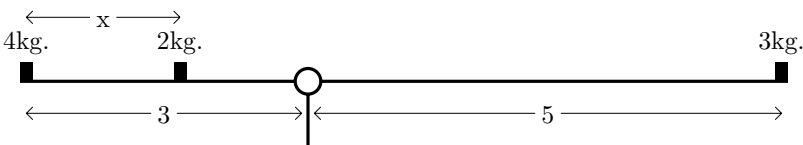
2.(5 pts.)  $\frac{d}{dx} \ln(\ln x) = ?$

- (a)  $\frac{1}{\ln x}$  (b)  $\frac{\ln x}{x}$  (c)  $\frac{\ln x}{\ln(\ln x)}$  (d)  $\frac{1}{x \ln x}$  (e)  $\frac{x}{\ln x}$

3.(5 pts.)  $\int_0^{\pi/3} \tan x \, dx =$

- (a) 4 (b) 3 (c)  $\ln \frac{1}{2}$  (d)  $\sqrt{3}$  (e)  $\ln 2$

4.(5 pts.) On the teeter-totter with weights as indicated you need to place one further mass of 2kg in such a way as to balance the beam. At what distance from the *left*-end is the correct location?



- (a) 0.5 (b) 1.0 (c) 1.5 (d) 2.0  
 (e) There is no such distance.

5.(5 pts.) Suppose you know the following results about a differentiable function:

- its inverse function  $g$  exists
- $f(2) = 4$  and  $f'(2) = 3$ . Which of the following is certain to be true?

- (a)  $g'(4) = \frac{1}{3}$  (b)  $g'(3) = \frac{1}{4}$  (c)  $g'(4) = \frac{1}{2}$  (d)  $g'(2) = \frac{3}{4}$  (e)  $g'(2) = \frac{1}{3}$

6.(5 pts.) If  $y = \frac{1}{2}(e^x + e^{-x})$ , then  $\int_{-2}^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx =$

- (a)  $e^2$  (b)  $e^2 - e^{-2}$  (c)  $e^2 - 1$  (d)  $e^2 + e^{-2}$  (e)  $2e^2$

7.(5 pts.)  $\frac{d}{dx} 4 \arctan e^x \Big|_{x=0} =$

- (a) 0                      (b) 1                      (c) 2                      (d) 3                      (e) 4

8.(5 pts.) Which of the following is an integrating factor for the expression

$$y' + (\cot x)y$$

i.e. which function  $v$  below satisfies  $(vy)' = vy' + v(\cot x)y$ .

- (a) 1                      (b)  $\sin y$                       (c)  $e^{\sin x}$                       (d)  $\ln \sin x$                       (e)  $\sin x$

9.(5 pts.) Which of the following statements is true for general  $x > 0$

- (a)  $10^{\ln x} = x^{\log_{10} e}$                       (b)  $10^{\ln x} = x^{\ln 10}$                       (c)  $10^{\ln x} = \ln 10^x$   
 (d)  $10^{\ln x} = e^{\log_{10} x}$                       (e)  $10^{\ln x} = (\ln x)^{10}$

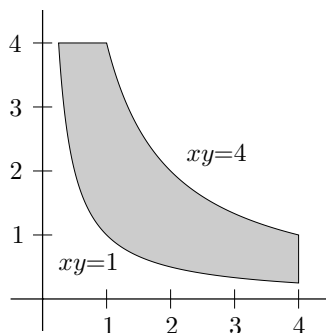
10.(5 pts.)

$$\frac{d}{dx} (\arccos x)^2 =$$

- (a)  $\frac{-2 \arccos x}{\sqrt{1-x^2}}$                       (b)  $\frac{-2x}{\sqrt{1-x^2}}$                       (c)  $\arccos \frac{x}{\sqrt{1-x^2}}$                       (d)  $\frac{\sin x}{\cos^2 x}$   
 (e)  $\frac{-2x}{\sqrt{1-x^4}}$

### Partial Credit

11.(13 pts.)



- (a) Find the area of the boomerang shaped (shaded) region.  
 (b) Find the moment about the axis of your choice.  
 (c) Use symmetry to find the coordinates of the center of mass.

**Hint for (a) and (b):** The integral for each answer will be in two pieces.

12.(13 pts.) Evaluate the integral  $\int_{-1}^0 \frac{1}{x^2 + 2x + 2} dx$

13.(12 pts.) Find  $y(1)$  if  $y$  is the unique function which satisfies

$$y' = \sqrt{1 - y^2} \cdot x \quad , \quad y(0) = 0$$

14.(12 pts.) Find the general solution to the differential equation

$$y' = \frac{2y}{x} + x \quad (x > 0)$$

Name: ANSWERS

Instructor: ANSWERS

Exam I  
February 15, 2001

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

- |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 1.  | (a) | (b) | (c) | (●) | (e) |
| 2.  | (a) | (b) | (c) | (●) | (e) |
| 3.  | (a) | (b) | (c) | (d) | (●) |
| 4.  | (a) | (b) | (●) | (d) | (e) |
| 5.  | (●) | (b) | (c) | (d) | (e) |
| 6.  | (a) | (●) | (c) | (d) | (e) |
| 7.  | (a) | (b) | (●) | (d) | (e) |
| 8.  | (a) | (b) | (c) | (d) | (●) |
| 9.  | (a) | (●) | (c) | (d) | (e) |
| 10. | (●) | (b) | (c) | (d) | (e) |

DO NOT WRITE IN THIS BOX!

Total multiple choice: \_\_\_\_\_

11. \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

14. \_\_\_\_\_

**Total:** \_\_\_\_\_