Multiple Choice

1.(6 pts.) What can be said about the improper integral

$$\int_1^{\infty} xe^{-x}dx?$$

(a) It converges to e^{-1}. (b) It diverges. (c) It converges to π.

(d) It converges to 1. (e) It converges to $2e^{-1}$.

2.(6 pts.) The *sequence* given by $a_n = (\frac{n}{n+1})^n$

(a) converges to π. (b) converges to 1. (c) diverges.

(d) converges to e^{-1}. (e) converges to e^2.

3.(6 pts.) The series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{5^{n+2}}$

(a) converges to $\frac{1}{50}$. (b) converges to π. (c) diverges.

(d) converges to 2. (e) converges to $\frac{1}{10}$.

4.(6 pts.) The sequence $a_n = \dfrac{2^n}{(n+1)!}$ for $n \geq 1$ is

(a) nondecreasing and convergent.

(b) nonincreasing and convergent.

(c) nonincreasing and divergent.

(d) neither nonincreasing nor nondecreasing.

(e) nondecreasing and divergent.

5.(6 pts.) The series $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

(a) converges to 1. (b) diverges. (c) converges to π.

(d) converges to 25. (e) converges to e.

6.(6 pts.) Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series with nonnegative terms. Which of the following is always a correct statement?

(a) If $\sum_{n=1}^{\infty} a_n$ diverges and $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty}(a_n - b_n)$ converges.

(b) If $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} \frac{a_n}{b_n}$ converges provided all $b_n \neq 0$.

(c) If $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty}(a_n + b_n)$ diverges.

(d) If $\sum_{n=1}^{\infty} a_n$ diverges and $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty} \frac{a_n}{b_n}$ diverges.

(e) None of the above.

7.(6 pts.) For which values of x does the power series

$$\sum_{n=1}^{\infty} (\ln n) \left(\frac{x}{2}\right)^n$$

converge?

(a) $-1 < x < 1$. (b) all values of x. (c) $x = 0$ only. (d) $-2 < x < 2$.

(e) $\frac{-1}{\ln 2} < x < \frac{1}{\ln 2}$.

8.(6 pts.) The series

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

(a) diverges because $\lim_{n\to\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \neq 0$.

(b) converges absolutely.

(c) diverges because the terms alternate.

(d) diverges even though $\lim_{n\to\infty} \frac{(-1)^{n+1}}{\sqrt{n}} = 0$.

(e) does not converge absolutely but does converge conditionally.

Partial Credit

9.(13 pts.) Does the series

$$\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^{2n})}$$

converge or diverge? Show your reasoning and state clearly any theorems or tests you are using.

10.(13 pts.) Does the integral

$$\int_1^{\infty} \frac{dx}{\sqrt{x}(1+x)}$$

converge or diverge? Show all of your work and state clearly and precisely any theorems you are using.

11.(13 pts.) Find the **interval** of convergence of the series $\sum_{n=1}^{\infty} \frac{x^{2n}}{2^n\, n^2}$. Be sure to check the end points.

12.(13 pts.)

(a) Show that

$$\sum_{n=0}^{\infty}(-1)^n x^{2n} = \frac{1}{1+x^2}$$

provided that $|x| < 1$.

(b) Find

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)(\sqrt{3})^{2n+1}}.$$

(**Hint:** First use term-by-term integration on the series in part (a).)

Name: ANSWERS

Instructor: ANSWERS

Math 126
Exam III
April 24, 2001

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 9 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(•)
2.	(a)	(b)	(c)	(•)	(e)
3.	(•)	(b)	(c)	(d)	(e)
4.	(a)	(•)	(c)	(d)	(e)
5.	(a)	(•)	(c)	(d)	(e)
6.	(a)	(b)	(•)	(d)	(e)
7.	(a)	(b)	(c)	(•)	(e)
8.	(a)	(b)	(c)	(d)	(•)

DO NOT WRITE IN THIS BOX!

Total multiple choice:	
9.	
10.	
11.	
12.	
Total:	