1.(6 pts.) Let $f(x) = 2x^2 + 1$ for $x \geq 0$. Then $(f^{-1})'(9) =$

(a) $\frac{1}{9}$ (b) $\frac{1}{6}$ (c) $\frac{2}{9}$ (d) $\frac{1}{8}$ (e) $\frac{1}{24}$

2.(6 pts.) $\int_e^{e^2} \frac{1}{x \ln x} dx =$

(a) $\sum_{n=0}^{\infty} e^{2n}$ (b) $2e^2 - e$ (c) 2 (d) $\frac{1}{2e^2} - \frac{1}{e}$ (e) $\ln 2$

3.(6 pts.) If $y'(t) = te^{-t^2}$ and $y(0) = 1$, then $y(1) =$

(a) $\frac{1}{e^2} - \frac{1}{2}$ (b) $e - \frac{1}{e+1}$ (c) $e^3 - \frac{1}{6}$ (d) $\frac{3}{2} - \frac{1}{2e}$ (e) $e - \frac{1}{3}$

4.(6 pts.) $\lim_{x \to 0^+} \frac{\ln(e^x - 1)}{\ln(2x)} =$

Hint: Apply l'Hôpital's rule, simplify, and apply l'Hôpital's rule again.

(a) 2 (b) ∞ (c) 1 (d) -1 (e) 0

5.(6 pts.) Which of the following five functions

1) $\log_2(x+1)$, 2) $\ln(x+1)$, 3) $\ln(x^{100})$, 4) $\sqrt{x^2+1}$, 5) $\sqrt{\frac{x}{100}}$

grow at the same rate?

(a) Only 4) and 5) (b) Only 1) and 2) (c) Only 1), 2), and 3)

(d) Only 2) and 4) (e) 2) and 3), as well as 4) and 5)

6.(6 pts.) $\int_0^1 \frac{dx}{(1+x^2)^{\frac{3}{2}}} =$

Hint: This problem is shorter if you change limits of integration when you do the substitution.

(a) Diverges (b) $-\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$ (e) 1

7.(6 pts.) $\int_0^1 (x+1)e^{x+1} dx =$

(a) e (b) e^2 (c) $2e$ (d) Diverges (e) $2e^2$

8.(6 pts.) The solution to the initial value problem

$$x\frac{dy}{dx} + x^2y - x^2 = 0 \qquad y(0) = 0$$

is

(a) $y = 1 - e^{-\frac{x^2}{2}}$ (b) $y = e^{-x} - 1$

(c) $y = xe^x$ (d) $y = 1 - e^{-x}$

(e) $y = e^{-\frac{x^2}{2}} - 1$

9.(6 pts.) $\sec\big(\arccos\frac{1}{2}\big) =$

(a) 2 (b) 1 (c) 0 (d) $\frac{1}{2}$ (e) ∞

10.(6 pts.) Find $\int_{-1}^{1} \frac{dx}{x^3}$.
Be careful.

(a) -1 (b) 2 (c) diverges (d) 0 (e) 1

11.(6 pts.) Find $\sum_{n=1}^{\infty} \frac{2^{2n}}{5^{n+1}}$

(a) $\frac{5}{3}$ (b) $\frac{4}{5}$ (c) $\frac{1}{25}$ (d) $\frac{1}{2}$ (e) $\frac{1}{3}$

12.(6 pts.) Which of the following series converge?

(1) $\sum_{n=0}^{\infty} \frac{4^n}{n!}$ (2) $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^2}$ (3) $\sum_{n=1}^{\infty} \frac{n^2}{n^{\frac{8}{3}}}$

(a) (3) converges, (1) and (2) diverge (b) (2) and (3) converge, (1) diverges

(c) (1) converges, (2) and (3) diverge (d) (1) and (3) converge, (2) diverges

(e) (1) and (2) converge, (3) diverges

13.(6 pts.) Find $\lim_{n\to\infty} (1 - \frac{1}{2n})^n$

(a) e^2 (b) $\sqrt{e}$ (c) 1 (d) e^{-2} (e) $\frac{1}{\sqrt{e}}$

14.(6 pts.) Test the following series for absolute convergence, conditional convergence or divergence:

$$(1)\ \sum_{n=1}^{\infty}\frac{(-1)^n}{n^{0.9}} \qquad (2)\ \sum_{n=0}^{\infty}\frac{(-1)^n}{(1.1)^n} \qquad (3)\ \sum_{n=0}^{\infty} n!(-1)^n$$

(a) (1) and (2) converge conditionally, (3) diverges

(b) (1) converges conditionally, (2) converges absolutely, (3) diverges

(c) (1) and (2) converge absolutely, (3) diverges

(d) (1) converges absolutely, (2) converges conditionally, (3) diverges

(e) (1) diverges, (2) and (3) converge conditionally.

15.(6 pts.) What is the MacLaurin series for xe^{x^2}?

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n x^{2n}$ (b) $\displaystyle\sum_{n=0}^{\infty} x^{2n+1}$ (c) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{n!}$

(d) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n}}{n}$ (e) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n+1}}{n!}$

16.(6 pts.) Find the interval of convergence for

$$\sum_{n=1}^{\infty}\frac{(-1)^n n x^n}{n^2+2}$$

(a) $-1 < x \leq 1$ (b) $-1 \leq x \leq 1$ (c) $-1 \leq x < 1$ (d) $-1 < x < 1$ (e) all x

17.(6 pts.) Find the first 3 non–zero terms of the MacLaurin series solution to

$$y' + xy = 0 \qquad y(0) = 1$$

(a) $1 - \frac{1}{2}x + \frac{1}{8}x^2$ (b) $1 + x + x^2$ (c) $x - \frac{1}{2}x^3 + \frac{1}{8}x^5$

(d) $x + x^3 + x^5$ (e) $1 - \frac{1}{2}x^2 + \frac{1}{8}x^4$

18.(6 pts.) Which line below is the directrix of the parabola $y^2 = 8x$.

(a) $y = -2$ (b) $x = -2$ (c) $y = 2$

(d) $x = 2$ (e) Can not be determined from the given information

19.(6 pts.) What is the equation of the ellipse with vertices $(\pm 4, 0)$ and eccentricity $e = \dfrac{1}{2}$?

(a) $\dfrac{x^2}{12} + \dfrac{y^2}{16} = 1$ (b) $\dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$ (c) $\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$

(d) $\dfrac{x^2}{12} + \dfrac{y^2}{4} = 1$ (e) $\dfrac{x^2}{16} + \dfrac{y^2}{12} = 1$

20.(6 pts.) Find the slope of the parameterized curve

$$x^3 + t^3 + 2t = 8 \qquad e^y = t + \cos t$$

when $t = 0$.

Hint: First find $x(0)$ and $y(0)$ and then use implicit differentiation.

(a) -6 (b) -3 (c) 3 (d) 0 (e) 6

21.(6 pts.) Find the arclength of the parameterized curve

$$x(t) = 1 - \cos t \qquad y(t) = 4 - \sin t \qquad -2 \le t \le 3\ .$$

(a) 2π (b) 3 (c) 5 (d) 4 (e) 2

22.(6 pts.) Which formula below yields the surface area of the surface obtained by rotating the paramterized curve

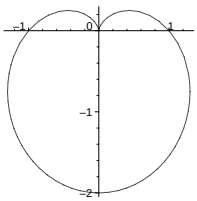
$$x(t) = \frac{t^2}{2} \quad y(t) = \frac{t^3}{3} \qquad 0 \le t \le 1$$

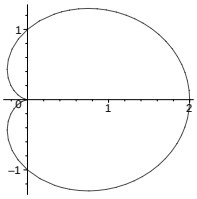
about the y–axis?

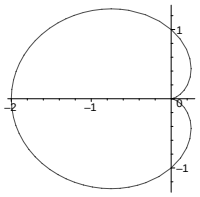
(a) $2\pi \displaystyle\int_0^1 \frac{t^3\sqrt{t^2+1}}{3}\,dt$ (b) $2\pi \displaystyle\int_0^1 \frac{t^2\sqrt{t^2+1}}{2}\,dt$ (c) $2\pi \displaystyle\int_0^1 \frac{t^2\sqrt{t^2+1}}{3}\,dt$

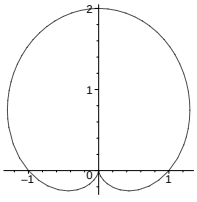
(d) $2\pi \displaystyle\int_0^1 \frac{t^3\sqrt{t^2+1}}{2}\,dt$ (e) $2\pi \displaystyle\int_0^1 \frac{t^4\sqrt{t^2+1}}{4}\,dt$

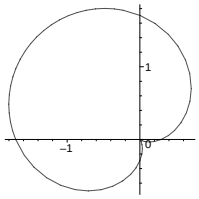
23.(6 pts.) Which graph below is the graph of the polar curve $r = 1 - \sin(\theta)$?

(a) 

(b) 

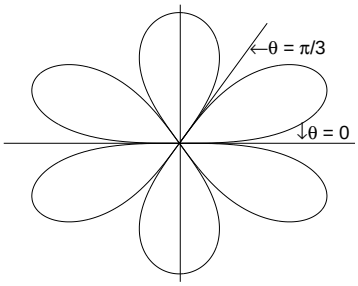
(c) 

(d) 

(e) 

24.(6 pts.) Polar area Find the area inside the six–leafed rose $r^2 = 2\sin 3\theta$.

Warning: Be careful with your limits of integration.



(a) 1 (b) 2 (c) 0 (d) 4 (e) 3

25.(6 pts.) Find the arclength of the curve given in polar coordinates by $r = \cos\theta + \sin\theta$ for $0 \leq \theta \leq \pi$.

(a) $\frac{\pi}{2}$ (b) $\sqrt{2}\pi$ (c) $\frac{7\pi}{8}$ (d) π (e) $\sqrt{3}\pi$

Name: _____

Instructor: Bullwinkle

Math 126, Final

May 12, 2000

- The Honor Code is in effect for this examination. All work is to be your own.
- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- You are to hand in just the front page.

Good Luck!

Please mark your answers with an **X**! Do NOT circle them!

The dotted lines in the answer box indicate page breaks.

1.	(a)	(b)	(c)	(•)	(e)	15.	(a)	(b)	(c)	(d)	(•)
2.	(a)	(b)	(c)	(d)	(•)	16.	(•)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(•)	(e)	17.	(a)	(b)	(c)	(d)	(•)
4.	(a)	(b)	(•)	(d)	(e)	18.	(a)	(•)	(c)	(d)	(e)
5.	(a)	(b)	(•)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(•)
6.	(a)	(b)	(c)	(•)	(e)	20.	(•)	(b)	(c)	(d)	(e)
7.	(a)	(•)	(c)	(d)	(e)	21.	(a)	(b)	(•)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	(•)	(e)
9.	(•)	(b)	(c)	(d)	(e)	23.	(•)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(•)	(d)	(e)	24.	(a)	(b)	(c)	(•)	(e)
11.	(a)	(•)	(c)	(d)	(e)	25.	(a)	(•)	(c)	(d)	(e)
12.	(a)	(b)	(•)	(d)	(e)						
13.	(a)	(b)	(c)	(d)	(•)	Score: _____					
14.	(a)	(•)	(c)	(d)	(e)						