Name: ______________________________

Instructor: ______________________________

Math 126, Final

May 7, 2002

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- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- You are to hand in just the front page.

Good Luck!

Please mark your answers with an **X**! Do NOT circle them!

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1.(6 pts.) Let $f(x) = e^x - 1$ and let f^{-1} denote the inverse function. Then $(f^{-1})'(e^2 - 1) = ?$

(a) $\dfrac{1}{e^2-1}$ (b) e (c) e^2 (d) e^{-1} (e) e^{-2}

2.(6 pts.) $\displaystyle\int_{\tan(1)}^{\tan(2)} \frac{\arctan^5(x)}{1+x^2}\,dx = ?$

(a) Diverges (b) $\dfrac{21}{2}$ (c) $\dfrac{32}{3}$ (d) $\dfrac{\tan(3)}{6}$ (e) $\dfrac{11}{2}$

3.(6 pts.) $\int_1^2 \arctan(x)\,dx = ?$

(a) $2\arctan(1) - \arctan(2) + \dfrac{\ln 5 - \ln 2}{2}$

(b) $\dfrac{\pi}{4} - \ln(5/2)$

(c) $\sqrt{\dfrac{\pi}{2}} \cdot \dfrac{3}{2}$

(d) $2\arctan(2) - \arctan(1) + \dfrac{\ln 2 - \ln 5}{2}$

(e) $\ln\big(\arctan(2)\big) - \arctan\big(\ln(5)\big)$

4.(6 pts.) $\lim_{x\to 0^+} (\cos x)^{\frac{1}{x^2}} =$

(a) $e^{-\frac{1}{2}}$

(b) 1

(c) e

(d) ∞

(e) Does not exist

5.(6 pts.) The moment about the y–axis of the plane region above the parabola $y = x^2$ and below the line $y = 9$ is given by the integral

(a) $\displaystyle\int_0^9 \frac{y^2}{2}\,dy$ (b) $\displaystyle 4\int_0^3 x(9-x^2)\,dx$ (c) $\displaystyle\int_{-3}^3 x(9-x^2)\,dx$

(d) $\displaystyle\int_{-3}^3 \frac{(9-x^2)^2}{2}\,dx$ (e) $\displaystyle\int_0^9 y(9-y)\,dy$

6.(6 pts.) The solution to the initial value problem

$$x\frac{dy}{dx} + 2y = e^{x^2} \qquad y(1) = 0$$

is

(a) $y = \dfrac{e^x - e}{2x^2}$ (b) $y = \dfrac{e^{x^2} - e}{2x^2}$ (c) $y = \dfrac{e^{x^2} - e}{2x}$

(d) $y = \dfrac{e^x - e}{2x}$ (e) $y = xe^x - e$

7.(6 pts.) The solution to the initial value problem

$$y' = x\cos y \qquad\qquad y(2) = 0$$

satisfies the implicit equation

(a) $\cos(y) = x + \cos(2)$

(b) $\cos y = x - 1$

(c) $e^{2y+1} = \arcsin(x-2) + e$

(d) $\ln|\sec(y) + \tan(y)| = \frac{x^2}{2} - 2$

(e) $\frac{ey}{2} = e^{\cos x} - e^{\cos 2}$

8.(6 pts.) Which integral below gives the arclength of the curve $x = 1 - 2\cos t$, $y = \sin^2(t/2)$, $0 \le t \le 2\pi$?

(a) $\int_0^{2\pi} \sqrt{4\sin^2 t + \sin^4(t/2)}\ dt$

(b) $\int_0^{2\pi} \sqrt{1 - 2\cos(t) + \cos^2(t) + \sin^2(t/2)\cos^2(t/2)}\ dt$

(c) $\int_0^{2\pi} \sqrt{4\sin^2 t + \sin^2(t/2)\cos^2(t/2)}\ dt$

(d) $\int_0^{2\pi} \sqrt{\sin^2(t/2) - 2\sin^2(t/2)\cos(t)}\ dt$

(e) $\int_0^{2\pi} \sqrt{1 - 2\cos(t) + \cos^2(t) + \sin^4(t/2)}\ dt$

9.(6 pts.) $\int_0^2 \frac{dx}{1-x} =$

(a) $\frac{\pi}{6}$ (b) Diverges (c) $\frac{\pi}{\sqrt{2}}$ (d) $\ln 2$ (e) 0

10.(6 pts.) $\int_0^{\pi/2} x\cos(x)dx =$

(a) $\frac{\pi}{2} - 1$ (b) $\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}$ (c) Diverges (d) 0 (e) $1 - \frac{\pi}{2}$

11.(6 pts.) You begin an experiment at 9am with a sample of 1000 bacteria. An hour later your population has doubled. Assuming exponential growth, what is the population at noon?

(a) 4000 (b) $1000e^3$ (c) $1000e^{-3}$ (d) 32000 (e) 8000

12.(6 pts.) If you expand $\dfrac{2x+1}{x^3+x}$ as a partial fraction, which expression below would you get?

(a) $\dfrac{-1}{x^2} + \dfrac{1}{x+1}$ (b) $\dfrac{-1}{x} + \dfrac{x}{x^2+1}$ (c) $\dfrac{1}{x} + \dfrac{-x+2}{x^2+1}$

(d) $\dfrac{-2}{x} + \dfrac{1}{x^2+1}$ (e) $\dfrac{2}{x} + \dfrac{1}{x^2+1}$

13.(6 pts.) The point $\left(2, \dfrac{11\pi}{3}\right)$ in polar coordinates corresponds to which point below in Cartesian coordinates?

(a) $(-1, \sqrt{3})$ (b) $(-\sqrt{3}, 1)$ (c) $(\sqrt{3}, -1)$ (d) $(1, -\sqrt{3})$

(e) Since $\dfrac{11\pi}{3} > 2\pi$, there is no such point

14.(6 pts.) Which integral below gives the surface area of the surface of revolution obtained by rotating the polar curve $r = \cos\theta$, $0 \le \theta \le \pi$ about the y–axis?
Hint: A polar curve is also a parameterized curve.

(a) $2\pi \displaystyle\int_0^\pi \cos^2\theta \, d\theta$ (b) $2\pi \displaystyle\int_0^\pi \sin\theta \, \cos\theta \, d\theta$ (c) $\dfrac{\pi}{2} \displaystyle\int_0^\pi \sin^2\theta \, \cos\theta \, d\theta$

(d) $2\pi \displaystyle\int_0^\pi \sin^2\theta \, \cos\theta \, d\theta$ (e) $2\pi \displaystyle\int_0^\pi \sin^2\theta \, d\theta$

15.(6 pts.) Find the area inside the cardioid $r = 1 + \sin\theta$.

cardioid.eps

(a) $3\pi + \ln 4$ (b) 2 (c) $\frac{3}{2}$ (d) 2π (e) $\frac{3\pi}{2}$

16.(6 pts.) Which point below is a focus for the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$?

(a) $(4, 0)$ (b) $(0, 5)$ (c) $(3, 0)$ (d) $(0, 4)$

(e) $(-5, 0)$

17.(6 pts.) Find $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}}{3 \cdot 5^{n-1}}$

(a) $\frac{5}{3}$ (b) $\frac{20}{3}$ (c) $\frac{4}{15}$ (d) $\frac{5}{4}$ (e) $\frac{5}{12}$

18.(6 pts.) Which of the following series converge conditionally?

(1) $\displaystyle\sum_{n=0}^{\infty} \frac{\sin(2n)}{n!}$ (2) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n\, n}{(\ln n)^2}$ (3) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^3+1}$

(a) (2) and (3) converge conditionally, (1) does not converge conditionally

(b) (1) and (3) converge conditionally, (2) does not converge conditionally

(c) (2) converges conditionally, (1) and (3) do not converge conditionally

(d) (3) converges conditionally, (1) and (2) do not converge conditionally

(e) (1) and (2) converge conditionally, (3) does not converge conditionally

19.(6 pts.) Find the interval of convergence for

$$\sum_{n=1}^{\infty} \frac{x^n}{\sqrt[3]{n^2+2}}$$

Remark: $\dfrac{1}{\sqrt[3]{n^2+2}}$ is decreasing for $n > 0$.

(a) $-1 \le x \le 0$ (b) $-1 \le x \le 1$ (c) $-1 < x < 1$ (d) $-1 < x \le 1$ (e) $-1 \le x < 1$

20.(6 pts.) If $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{(2n+1)!}$, find the power series centered at 2 for the function $\displaystyle\int_2^x f(t)\, dt$.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n^2)(2n+1)!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n+1)!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n+1)(2n+1)!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{2n+1}}{(n+1)(2n)!}$

(e) The given function can not be represented by a power series centered at 2.

21.(6 pts.) Which series below is the MacLaurin series (Taylor series centered at 0) for $\dfrac{x^2}{1+x}$?

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\ x^{n+2}$

(b) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{n+2}}{n+2}$

(c) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n x^{2n-2}}{n!}$

(d) $\displaystyle\sum_{n=0}^{\infty}x^{2n+2}$

(e) $\displaystyle\sum_{n=0}^{\infty}(-1)^n x^{2n}$

22.(6 pts.) Find the degree 3 MacLaurin polynomial (Taylor polynomial centered at 0) for the function

$$\frac{e^x}{1-x^2}$$

(a) $1+x+\dfrac{x^2}{6}+0\,x^3$

(b) $1+x-\dfrac{5x^3}{3}$

(c) $1+x+\dfrac{3x^2}{2}+\dfrac{7x^3}{6}$

(d) $1+x-\dfrac{x^3}{6}$

(e) $1-\dfrac{x^2}{2}+\dfrac{x^3}{5}$

23.(6 pts.) $\lim_{x \to 0} \dfrac{\sin(x^3) - x^3}{x^9} =$

Hint: Without MacLaurin series this may be a long problem.

(a) $\dfrac{9}{7}$ (b) $-\dfrac{1}{6}$ (c) $\dfrac{7}{9}$ (d) 0 (e) ∞

24.(6 pts.) Which series below is a power series for $\cos(\sqrt{x})$?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n+1)!}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{x}^{\,n}}{(2n)!}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$ (d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n-\frac{1}{2}}}{(2n)!}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^2+1}$

25.(6 pts.) What is the radius of convergence for the binomial series for the function $\dfrac{1}{\sqrt[5]{3-x}}$?

(a) $\sqrt[5]{3}$ (b) $\dfrac{3}{5}$ (c) 1 (d) 3 (e) $\dfrac{2}{3}$

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