Multiple Choice

1.(5 pts.) Let $x = \sin(9t)$ and $y = \cos(9t)$. Then $\dfrac{dy}{dx} = ?$

(a) $81\sec^2(9t)$ (b) $-\tan(9t)$ (c) $9\tan(t)$ (d) $\tan(9t)$ (e) $\cot(9t)$

2.(5 pts.) Find the points on the curve $x = t^2$, $y = t^2 - 2t$ where the tangent line is horizontal.

(a) $(x, y) = (2, 1)$ (b) $(x, y) = (1, 1)$ (c) $(x, y) = (1, -1)$

(d) $(x, y) = (1, 2)$ (e) $(x, y) = (2, -1)$

3.(5 pts.) Find the length of the curve $x = 4e^{\frac{t}{2}}$, $y = e^t - 1$, $0 \le t \le 1$.

(a) $\displaystyle\int_0^1 \sqrt{e^{2t} - 14e^t + 1}\ dt$ (b) $\displaystyle\int_0^1 \sqrt{1 + e^{2t}}\ dt$

(c) $\displaystyle\int_0^1 \sqrt{e^{2t} + 4e^t}\ dt$ (d) $\displaystyle\int_0^1 \sqrt{e^{2t} + 14e^t + 1}\ dt$

(e) $\displaystyle\int_0^1 \sqrt{1 + 4e^t}\ dt$

4.(5 pts.) Find the area of the surface obtained by rotating the curve $x = 3t^2$, $y = t^3 - 3t$, $0 \le t \le 1$ around the y–axis.

(a) 3 (b) $\dfrac{48}{5}\pi$ (c) 18π (d) 1

(e) 0

5.(5 pts.) Find $\displaystyle\sum_{n=0}^{\infty} 3\left(\frac{-2}{5}\right)^n$.

(a) $\dfrac{3}{5}$ (b) $\dfrac{6}{5}$ (c) $\dfrac{5}{3}$ (d) $\dfrac{15}{7}$ (e) $\dfrac{17}{5}$

6.(5 pts.) Consider the parabola with vertex $(0,0)$, axis the x–axis and passing through $(1,-4)$. Which point below is the focus of this parabola?

(a) $(4,0)$ (b) $(-4,0)$ (c) $(-2,0)$ (d) $(2,0)$ (e) $(1,-2)$

7.(5 pts.) Given the sequence $s_n = \dfrac{9n+1}{10n+9}$, compute $\lim\limits_{n\to\infty} s_n$.

(a) $\dfrac{9}{10}$ (b) 0 (c) $\dfrac{10}{9}$ (d) $+\infty$ (e) $\dfrac{1}{9}$

8.(5 pts.) Evaluate the sum $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n^{100}} - \frac{1}{(n+1)^{100}}\right)$

(a) $\dfrac{1}{100}$ (b) $+\infty$ (c) 0 (d) Divergent but bounded

(e) 1

9.(5 pts.) Let $s = \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$, $s_n = \displaystyle\sum_{i=1}^{n} \frac{1}{i^3}$ and $R_n = s - s_n$. Use the integral test to estimate R_n.

(a) $\dfrac{1}{2}\dfrac{1}{(n+1)^3} \le R_n \le \dfrac{1}{2}\dfrac{1}{n^3}$

(b) $\dfrac{1}{3}\dfrac{1}{(n+1)^3} \le R_n \le \dfrac{1}{3}\dfrac{1}{n^3}$

(c) $\dfrac{1}{(n+1)^3} \le R_n \le \dfrac{1}{n^3}$

(d) $\dfrac{1}{2}\dfrac{1}{(n+1)^2} \le R_n \le \dfrac{1}{2}\dfrac{1}{n^2}$

(e) $\dfrac{1}{(n+1)^2} \le R_n \le \dfrac{1}{n^2}$

10.(5 pts.) Use Comparison Tests to determine which **one** of the following series is divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+8}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}\left(\frac{1}{2}\right)^n$ (c) $\displaystyle\sum_{n=1}^{\infty} 7\left(\frac{5}{6}\right)^n$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}+1}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+100}$

11.(5 pts.) The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$ is an alternating series which satisfies the conditions of the alternating series test. Find the smallest number k on the list below so that the k-th partial sum is within $\dfrac{1}{1,000}$ of the actual sum.

(a) 20 (b) 10 (c) 15 (d) 25

(e) 50

12.(5 pts.) Given that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ it follows that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n)^2} = \frac{1}{4}\left(\sum_{n=1}^{\infty} \frac{1}{n^2}\right) = \frac{\pi^2}{24}$.

Then $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = 1 + \frac{1}{9} + \frac{1}{25} + \cdots = ?$

(a) $\ln 2$ (b) $\dfrac{3\pi^2}{8}$ (c) $\dfrac{2\pi^2}{9}$ (d) $\dfrac{\pi^2}{8}$ (e) $\dfrac{\pi^2}{9}$

Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(13 pts.) Determine whether the series

$$\sum_{n=1}^{\infty} (-1)^n \, \frac{n^2+1}{n^3-9}$$

is absolutely convergent, conditionally convergent or divergent. The correct answer with no justification is worth 2 points.

14.(13 pts.) Find the radius of convergence and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \, (x-3)^n$$

The correct answer with no justification is worth 2 points.

15.(14 pts.) Find the power series centered at the origin for $f(x) = \dfrac{1}{(2-x)^2}$.

Hint: $f(x)$ is related to the derivative of a series you should know.
The correct answer with no justification is worth 2 points.

Name: ANSWERS

Instructor: ANSWERS

Exam III
April 23, 2002

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(•)	(c)	(d)	(e)
2.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(•)	(d)	(e)
4.	(a)	(•)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(•)	(e)
6.	(•)	(b)	(c)	(d)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(•)
9.	(a)	(b)	(c)	(•)	(e)
10.	(a)	(b)	(c)	(d)	(•)
11.	(a)	(•)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(•)	(e)

DO NOT WRITE IN THIS BOX!

Total multiple choice: _____

13. _____

14. _____

15. _____

Total: _____