

Multiple Choice

1.(5 pts.) Find the solution to the differential equation with initial value:

$$x^2y' + xy = 3x^3 + 1 \quad y(1) = 0$$

(a) $y = e^x(x^2 - 3x + 2)$ (b) $y = x^2 + \frac{\ln x}{x} - \frac{1}{x}$ (c) $y = \ln(x + e^x - 1) - 1.$

(d) $y = \frac{3x^4}{4} + x - \frac{7}{4}$ (e) $y = e^x - e$

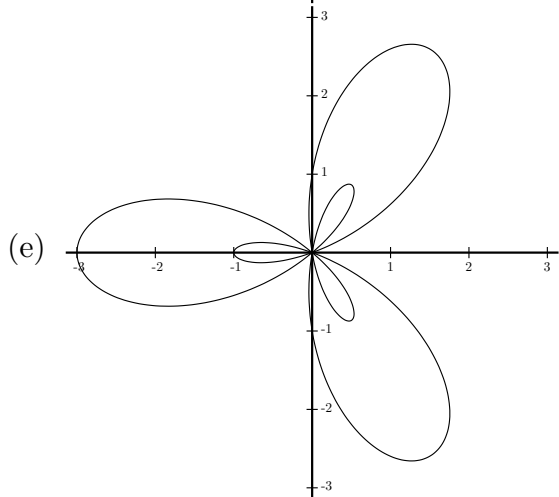
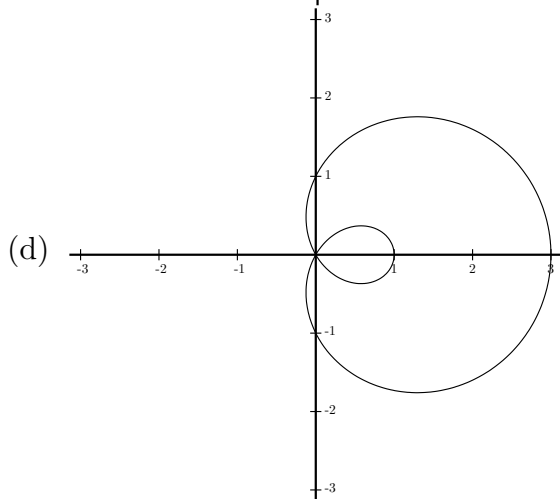
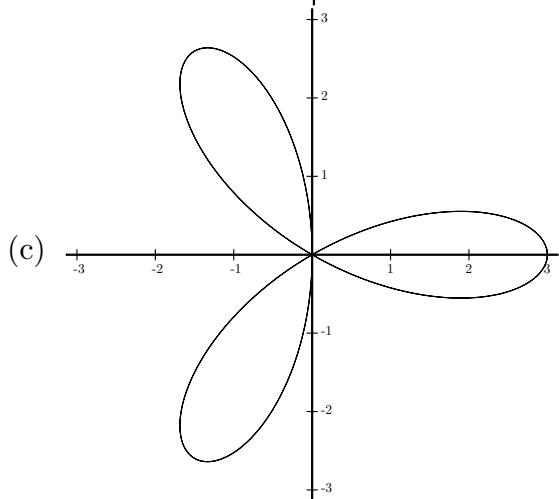
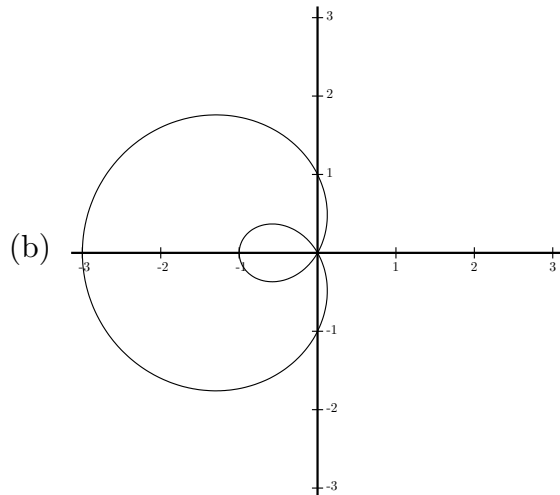
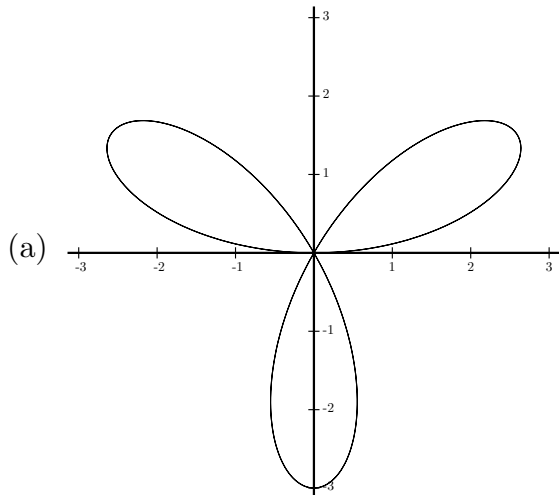
2.(5 pts.) Which integral below computes the area above the x -axis and below the parameterized curve $x(t) = t - \sin(t) + e^t$ and $y(t) = \sin(t)$ for $0 \leq t \leq \pi$.

(a) $\int_0^\pi \cos(t)(1 - \cos(t) + e^t) dt$ (b) $\int_0^\pi \cos(t)(1 - \sin(t) + e^t) dt$

(c) $\int_0^\pi \sin(t)(1 + 2\cos(t) + e^t) dt$ (d) $\int_0^\pi \sin(t)(1 - \sin(t) + e^t) dt$

(e) $\int_0^\pi \sin(t)(1 - \cos(t) + e^t) dt$

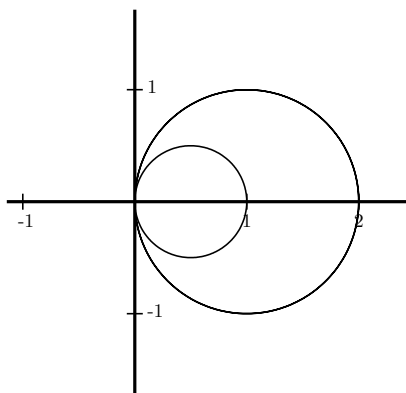
3.(5 pts.) Which graph below is the graph of the polar equation $r = 3 \sin(3\theta)$.



4.(5 pts.) Which integral below computes the length of the parameterized curve $x(t) = t + e^t$ and $y(t) = t^2 + e^t$ for $0 \leq t \leq \pi$.

- (a) $\int_0^\pi \sqrt{t^2 + 2te^t + 2e^{2t} + t^4 + 2t^2e^t} dt$ (b) $\int_0^\pi \sqrt{t^2 + 6te^t + 2e^{2t} + 4t^2} dt$
 (c) $\int_0^\pi \sqrt{1 + 2e^t + 2e^{2t} + t^4 + 2t^2e^t} dt$ (d) $\int_0^\pi \sqrt{1 + 2e^t + 2e^{2t} + 4t^2 + 4te^t} dt$
 (e) $\int_0^\pi \sqrt{1 + 2e^t - 4t^2 - 4te^t} dt$

5.(5 pts.) Which integral below is the area of the region inside the circle $r = 2 \cos \theta$ and outside the circle $r = \cos \theta$.



- (a) $\frac{1}{2} \int_0^\pi 5 \cos^2 \theta d\theta$ (b) $\frac{1}{2} \int_0^\pi 3 \cos^2 \theta d\theta$ (c) $\frac{1}{2} \int_0^\pi 4 \cos^2 \theta d\theta$
 (d) $\frac{1}{2} \int_0^\pi 3 \sin^2 \theta d\theta$ (e) $\frac{1}{2} \int_0^\pi 4 \sin^2 \theta d\theta$

6.(5 pts.) Find the length of the polar spiral $r = e^\theta$, $0 \leq \theta \leq 3$.

- (a) $\sqrt{2}e^3 - 1$ (b) $e^3 - 1$ (c) $e^3 + 1$
 (d) $\sqrt{2}(e^3 + 1)$ (e) $\sqrt{2}(e^3 - 1)$

7.(5 pts.) Which statement below is true about the series $\sum_{n=1}^{\infty} \frac{e^n}{n^2 + e^n}$

(a) $\lim_{n \rightarrow \infty} \frac{e^n}{n^2 + e^n}$ does not exist so the series converges.

(b) $\lim_{n \rightarrow \infty} \frac{e^n}{n^2 + e^n} = 0$ so the series diverges.

(c) $\lim_{n \rightarrow \infty} \frac{e^n}{n^2 + e^n} = 0$ so the series converges.

(d) $\lim_{n \rightarrow \infty} \frac{e^n}{n^2 + e^n} = 1$ so the series diverges.

(e) $\lim_{n \rightarrow \infty} \frac{e^n}{n^2 + e^n} = 1$ so the series converges.

8.(5 pts.) Sum the series $\sum_{n=2}^{\infty} \frac{2^n}{5^{2n}}$.

(a) $\frac{4}{23 \cdot 25}$

(b) $\frac{92}{25}$

(c) $\frac{5}{3}$

(d) $\frac{25}{23}$

(e) $\frac{100}{23}$

9.(5 pts.) Which of the statements below is true about the three series

$$\text{I) } \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \quad \text{II) } \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} \quad \text{III) } \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$$

(a) I) diverges, II) conditionally converges and III) absolutely converges.

(b) They all conditionally converge.

(c) I) absolutely converges, II) conditionally converges and III) diverges.

(d) I) conditionally converges; II) and III) absolutely converge

(e) They all absolutely converge.

10.(5 pts.) Consider the following two series:

$$\text{I) } \sum_{n=2}^{\infty} \frac{1}{n^3 - 1} \quad \text{and} \quad \text{II) } \sum_{n=2}^{\infty} \frac{1}{n^3} .$$

Which statement below is true?

- (a) Series II) converges but series I) diverges.
- (b) None of the other statements are true.
- (c) Both series converge.
- (d) Both series diverge.
- (e) Series I) converges but series II) diverges.

11.(5 pts.) Which series below conditionally converges?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^2}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} e^n}{\sqrt[5]{n^2}}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[5]{n^4}}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

(e) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[3]{n^4}}$

Partial Credit

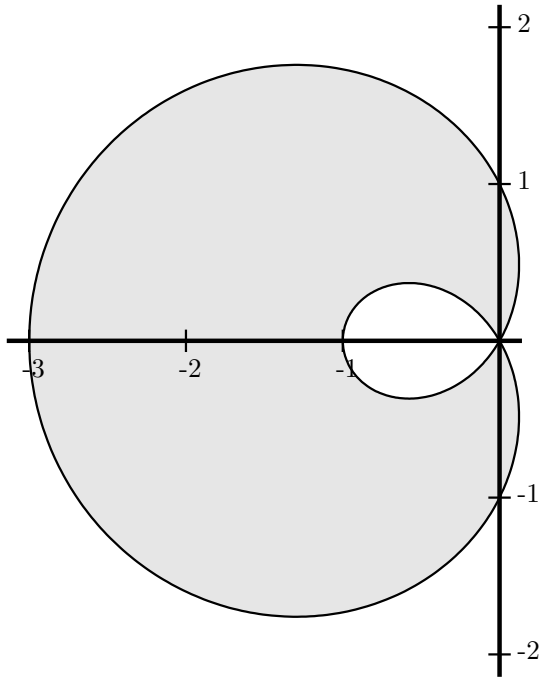
You must show your work on the partial credit problems to receive credit!

12.(15 pts.) Use the Integral Test to show that the series $\sum_{n=3}^{\infty} \frac{\ln n}{n}$ diverges.

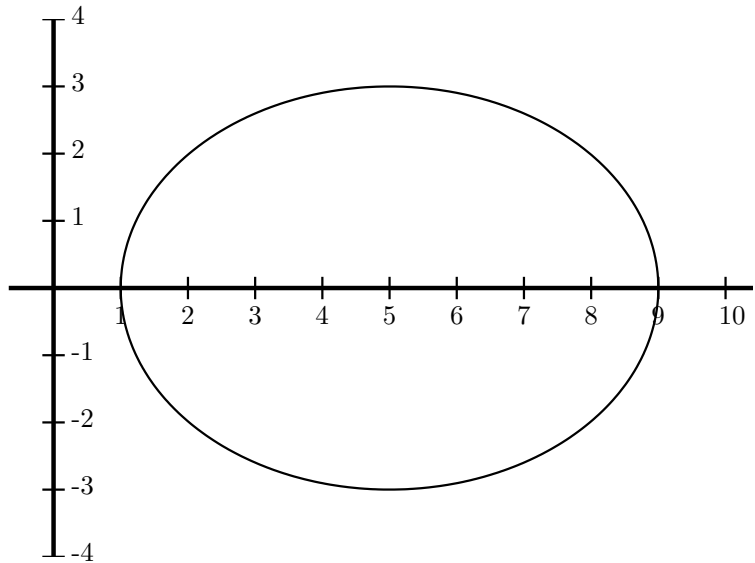
Remark: Be sure to check that the Integral Test can be applied.

13.(15 pts.) Set up an integral which computes the shaded area. The polar equation of the region is $r = 1 - 2 \cos \theta$.

Remarks: $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$. The answer is the difference of two areas.



14.(15 pts.) Set up an integral which computes the surface area of the surface obtained by rotating the parameterized ellipse $x(t) = 5 + 4\cos(t)$, $y(t) = 3\sin(t)$, $0 \leq t \leq 2\pi$ about the y -axis.



Name: ANSWERS

Instructor: ANSWERS

Math 126 Exam III
April 20, 2004

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 8 pages of the test.
- The backs of pages may be used if you need additional room to work on a problem.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (●) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (●) |
| 3. | (●) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (●) | (e) |
| 5. | (a) | (●) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (●) |
| 7. | (a) | (b) | (c) | (●) | (e) |
| 8. | (●) | (b) | (c) | (d) | (e) |
| 9. | (a) | (b) | (c) | (●) | (e) |
| 10. | (a) | (b) | (●) | (d) | (e) |
| 11. | (a) | (b) | (●) | (d) | (e) |

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Total multiple choice: _____

12. _____

13. _____

14. _____

Total: _____

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