

1.(6 pts.) Let $f(x) = x^7 - x^3 + 2x$. Find $(f^{-1})'(-2)$.

- (a) $1/6$ (b) $1/4$ (c) 4 (d) 1 (e) 6

2.(6 pts.) Solve the following equation for x .

$$7^x 5^x = 9e^2 .$$

- (a) $x = \frac{\ln 9 + 2}{\ln 12}$ (b) $x = \frac{2}{\ln 35}$ (c) $x = \frac{\ln 9 + 2}{\ln 35}$
(d) $x = \frac{2 \ln 7}{\ln 35}$ (e) There is no solution.

3.(6 pts.) Find the derivative of

$$y = x \cdot x^{1/x} .$$

(a) $x^{1/x} + x^{\frac{1}{x}-1}(1 - \ln x)$

(b) $x \cdot x^{\frac{1}{x}-3} = x^{\frac{1}{x}-2}$

(c) $x^{-1} \ln x(1 - \ln x)$

(d) $x^{1/x} + x^{\frac{1}{x}+1}(\ln(\ln x) - x^{-2})$

(e) $x^{1/x} + x^{\frac{1}{x}-2}$

4.(6 pts.) Evaluate the following limit.

$$\lim_{x \rightarrow \infty} x \cdot x^{1/x} .$$

Remark: Note the function is the same in both problems 3 and 4.

(a) 0

(b) e^{-1}

(c) ∞

(d) e

(e) 1

5.(6 pts.) Find $f'(x)$ for

$$f(x) = \ln(2^x + x) + \arctan(e^x)$$

(a) $\frac{2^x + 1}{2^x + x} + \frac{1}{\sqrt{1 - e^{2x}}}$

(b) $\frac{2^x \ln 2}{2^x + x} + \frac{e^x}{1 + e^{2x}}$

(c) $\frac{e^x \ln 2}{2^x + x} + \frac{e^x}{\sqrt{1 - 2e^x}}$

(d) $\frac{2^x \ln 2 + 1}{2^x + x} + \frac{e^x}{1 + e^{2x}}$

(e) $\frac{e^x \ln 2 + 1}{2^x + x} + \frac{e^x}{\sqrt{e^{2x} - 1}}$

6.(6 pts.) Which line below is the tangent line to the polar curve $r = 1 + \cos \theta$ at the point where $\theta = \frac{\pi}{2}$?

(a) $y = x + 1$

(b) $y = \frac{1 - \sin \theta}{1 + \cos \theta} (x + 1)$ (c) $x = 0$, a vertical tangent

(d) $y = \frac{\pi}{2}x + \frac{\pi}{2}$

(e) $y = 1$, a horizontal tangent

7.(6 pts.) Evaluate the following definite integral.

$$\int_1^e \frac{\ln x}{x^3} dx .$$

- (a) $\frac{1}{2}(1 - 2e^{-2})$ (b) $\frac{1}{4}(1 - 3e^{-2})$ (c) $\frac{1}{2}(e^{-2} - 1)$
(d) $\frac{1}{4}(e^{-3} + 2e^{-2} - 1)$ (e) $\frac{1}{4}(1 - e^{-3} + 2e^{-2})$

8.(6 pts.) Evaluate

$$\int \frac{x^2}{\sqrt{9 - x^2}} dx.$$

- (a) $\frac{1}{2}x\sqrt{9 - x^2} + C$ (b) $\frac{9}{2} \left[\arcsin(x/3) - \frac{x}{3} \right] + C$
(c) $\frac{9}{2} \left[\arcsin(x/3) - \frac{x\sqrt{9 - x^2}}{9} \right] + C$ (d) $\frac{9}{2} \left[\arcsin(x/3) - \frac{x^2}{9} \right] + C$
(e) $9 \arcsin(x/3) + C$

9.(6 pts.) Evaluate

$$\int \frac{x+7}{x^2+2x-3} dx.$$

(a) $\ln|x^2+2x-3|+C$

(b) $\ln\left|\frac{2(x+3)}{x-1}\right|+C$

(c) $\ln(2|x-1|-|x+3|)+C$

(d) $\ln\left|\frac{(x-1)^2}{x+3}\right|+C$

(e) $\ln\left|\frac{(x+3)^2}{x-1}\right|+C$

10.(6 pts.) Find the Midpoint Rule approximation (using four intervals) of

$$\int_{-1}^3 x^2 dx .$$

(a) 11

(b) $\frac{28}{3}$

(c) $\frac{45}{4}$

(d) 9

(e) $\frac{35}{4}$

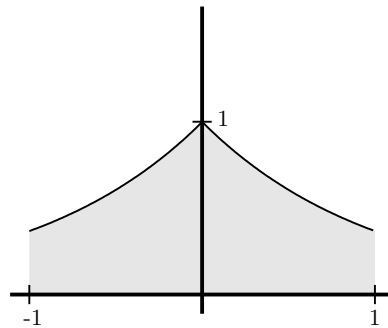
11.(6 pts.) Find the arclength of $y = 6 - 2x^{3/2}$, for $0 \leq x \leq 2$?

- (a) $\frac{1}{9}[\sqrt{19} - 1]$ (b) $\frac{2}{3}[19\sqrt{19}]$ (c) $\frac{20}{3}\sqrt{10}$
 (d) $\frac{2}{27}[10\sqrt{10} - 1]$ (e) $\frac{2}{27}[19\sqrt{19} - 1]$

12.(6 pts.) Find the center of mass of a plate with shape bounded below by the x -axis, on the left by $x = -1$, on the right by $x = 1$, and above by the curve

$$y = \begin{cases} 2^x & -1 \leq x \leq 0 \\ 2^{-x} & 0 \leq x \leq 1 \end{cases}.$$

The area is $\frac{1}{\ln 2}$.



- (a) $\left(0, \frac{3 - \ln 2}{5 + 2 \ln 2}\right)$ (b) $\left(0, \frac{3}{8 \ln 2}\right)$ (c) $\left(0, \frac{2 + \ln 2}{8}\right)$
 (d) $\left(0, \frac{1}{4}\right)$ (e) $\left(0, \frac{3}{8}\right)$

13.(6 pts.) Solve the initial value problem:

$$y' = (5 - 2x)(1 + y) \quad y(1) = 0.$$

(a) $y(x) = e^{x^2-5x} - e^{-2}$ (b) $y(x) = e^{-4+5x-x^2} - 1$ (c) $y(x) = e^{5x-x^2} - e^4$

(d) $y(x) = e^{-4+5x-x^2}$ (e) $y(x) = e^{-3+5x-x^2} - e$

14.(6 pts.) If 50 grams of radioactive material with a half-life of two days are present at day zero, how many grams are left at day five?

(a) $\frac{50}{\sqrt{32}}$ (b) $\frac{50}{\sqrt{8}}$ (c) $\frac{50}{32^{1/3}}$ (d) $\frac{50}{4^{1/3}}$ (e) $\frac{50}{10}$

15.(6 pts.) If the function $y(x)$ satisfies the differential equation

$$xy' - 2y = x - 2$$

subject to the initial value $y(1) = 1$, what is $y(3)$?

- (a) 3 (b) 4 (c) 7 (d) 5 (e) 6

16.(6 pts.) Which integral below computes the length of the parameterized curve $x(t) = t^3 - t$ and $y(t) = t^5 + t$ for $0 \leq t \leq 1$.

- (a) $\int_0^1 (t^3 - t)\sqrt{2 - 6t^2 + 19t^4 + 25t^8} dt$ (b) $\int_0^1 (t^5 + t)\sqrt{t^3 + t^5} dt$
(c) $\int_0^1 \sqrt{t^3 + t^5} dt$ (d) $\int_0^1 \sqrt{2 - 6t^2 + 19t^4 + 25t^8} dt$
(e) $\int_0^1 (t^5 + t)(3t^2 - 1) dt$

17.(6 pts.) Which integral below gives the area inside the polar curve $r = \cos(3\theta)$?

(a) $\frac{1}{2} \int_0^\pi \cos^2(3\theta) d\theta$

(b) $\frac{1}{2} \int_{\pi/6}^{\pi/3} \cos^2(3\theta) d\theta$

(c) $\frac{1}{2} \int_0^\pi \sqrt{\cos^2(3\theta) + 9 \sin^2(3\theta)} d\theta$

(d) $\frac{1}{2} \int_0^{2\pi} \cos^2(3\theta) d\theta$

(e) $\frac{1}{2} \int_0^{2\pi} \sqrt{\cos^2(3\theta) + 9 \sin^2(3\theta)} d\theta$

18.(6 pts.) Which statement below is true about the series $\sum_{n=1}^{\infty} \frac{3^n}{n^3 + 2 \cdot 3^n}$

(a) $\lim_{n \rightarrow \infty} \frac{3^n}{n^3 + 2 \cdot 3^n} = 0$ so the series converges.

(b) $\lim_{n \rightarrow \infty} \frac{3^n}{n^3 + 2 \cdot 3^n} = 1/2$ so the series converges.

(c) $\lim_{n \rightarrow \infty} \frac{3^n}{n^3 + 2 \cdot 3^n} = 1/2$ so the series diverges.

(d) $\lim_{n \rightarrow \infty} \frac{3^n}{n^3 + 2 \cdot 3^n}$ does not exist so the series converges.

(e) $\lim_{n \rightarrow \infty} \frac{3^n}{n^3 + 2 \cdot 3^n} = 0$ so the series diverges.

19.(6 pts.) Sum the series $\sum_{n=1}^{\infty} \frac{7^n}{5^{2n}}$.

(a) $\frac{25}{22}$

(b) $\frac{7}{18}$

(c) $\frac{5}{7}$

(d) $\frac{96}{25}$

(e) $\frac{3}{22}$

20.(6 pts.) The interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} (x+3)^n$$

is

(a) $[2, 4]$

(b) $(-1, 1)$

(c) $(-4, -2)$

(d) $(2, 4)$

(e) $(-4, -2]$

21.(6 pts.) Which series below absolutely converges?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)}$ (c) $\sum_{n=1}^{\infty} \frac{\sqrt{n^3}}{n^2+1}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \pi^n}{3^n}$

(e) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$

22.(6 pts.) Which series below conditionally converges? **These are the same series as in problem 21.**

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)}$ (c) $\sum_{n=1}^{\infty} \frac{\sqrt{n^3}}{n^2+1}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \pi^n}{3^n}$

(e) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$

23.(6 pts.) Find a power series for the function

$$f(x) = -\frac{3}{(x+1)^2}.$$

centered at 1.

(a) $\frac{3}{2} \sum_{n=1}^{\infty} (-1)^n n \frac{(x-1)^{n-1}}{2^n}$

(b) $\frac{3}{2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)} \frac{(x-1)^{n+1}}{2^n}$

(c) $\frac{3}{2} \sum_{n=1}^{\infty} n \frac{(x-1)^{n-1}}{2^n}$

(d) $\frac{3}{2} \sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^n}{2^n}$

(e) $\frac{3}{2} \sum_{n=1}^{\infty} (-1)^n n \frac{(x-1)^n}{2^n}$

24.(6 pts.) Which series below represents $\frac{\sin x - x}{x^3}$?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-2}}{(2n)!}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-2}}{n!}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-2}}{(2n+1)!}$

(d) $\sum_{n=1}^{\infty} (-1)^n \binom{1/2}{n} x^{2n-2}$

(e) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n)!}$

25.(6 pts.) Which series below gives the arc length of the curve $y = \frac{x^3}{3}$ from $x = 0$ to $x = 1$?

(a) $\sum_{n=0}^{\infty} \binom{1/2}{n} \frac{1}{4n+2}$ (b) $\sum_{n=0}^{\infty} \binom{1/3}{n} \frac{1}{4n+1}$ (c) $\sum_{n=0}^{\infty} \binom{1/3}{n} \frac{1}{4n+2}$

(d) $\sum_{n=0}^{\infty} \binom{1/2}{n} \frac{(-1)^n}{4n+1}$ (e) $\sum_{n=0}^{\infty} \binom{1/2}{n} \frac{1}{4n+1}$

Math 126
Final Exam
May 6, 2004

Name: _____

Instructor: _____

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
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Please mark your answers with an **X!** Do NOT circle them!

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1.	(a)	(b)	(c)	(d)	(e)	15.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(c)	(d)	(e)
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3.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
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5.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
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7.	(a)	(b)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	(d)	(e)
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9.	(a)	(b)	(c)	(d)	(e)	23.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(d)	(e)
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11.	(a)	(b)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(e)
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13.	(a)	(b)	(c)	(d)	(e)	Previous Total:	_____				
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Final Exam
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Name: _____

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