

1.(6 pts.) Let $f(x) = x^5 - x^3 + 2x$. Find $(f^{-1})'(2)$.

- (a) $1/4$ (b) $1/70$ (c) 4 (d) $1/5$ (e) 70

2.(6 pts.) Solve the following equation for x .

$$7^x 5^x = 5 e^2 .$$

- (a) $x = \frac{\ln 5 + 2}{\ln 12}$ (b) $x = \frac{2}{\ln 35}$ (c) $x = \frac{\ln 5 + 2}{\ln 35}$
(d) $x = \frac{2 \ln 7}{\ln 35}$ (e) There is no solution.

3.(6 pts.) Find the derivative of

$$y = x^{1/x} .$$

- (a) $x^{\frac{1}{x}-2}(1 - \ln x)$ (b) $-x^{\frac{1}{x}-3}$ (c) $x^{-1} \ln x(1 - \ln x)$
(d) $x^{\frac{1}{x}}(\ln(\ln x) - x^{-2})$ (e) $x^{\frac{1}{x}-2}$

4.(6 pts.) Evaluate the following limit.

$$\lim_{x \rightarrow \infty} x^{1/x} .$$

Remark: Note the function is the same in both problems 3 and 4.

- (a) 0 (b) e^{-1} (c) 1 (d) e (e) ∞

5.(6 pts.) Find $f'(x)$ for

$$f(x) = \ln(2^x + x) + \arcsin(e^x)$$

(a) $\frac{2^x + 1}{2^x + x} + \frac{1}{\sqrt{1 - e^{2x}}}$

(b) $\frac{2^x \ln 2}{2^x + x} + \frac{e^x}{1 + e^{2x}}$

(c) $\frac{e^x \ln 2}{2^x + x} + \frac{e^x}{\sqrt{1 - 2e^x}}$

(d) $\frac{2^x \ln 2 + 1}{2^x + x} + \frac{e^x}{\sqrt{1 - e^{2x}}}$

(e) $\frac{e^x \ln 2 + 1}{2^x + x} + \frac{e^x}{\sqrt{e^{2x} - 1}}$

6.(6 pts.) Which line below is the tangent line to the parameterized curve $x = t - \cos t$, $y = t + \sin t$ when $t = 0$?

(a) $y = 2x + 2$

(b) $y = \frac{t + \sin t}{t - \cos t} (x + 1)$ (c) $x = -1$, a vertical tangent

(d) $y = \frac{\pi}{2}x + \frac{\pi}{2}$

(e) $y = \frac{1 + \cos t}{1 + \sin t} (x + 1)$

7.(6 pts.) Evaluate the following definite integral.

$$\int_1^e \frac{\ln x}{x^3} dx .$$

- (a) $\frac{1}{2}(1 - 2e^{-2})$ (b) $\frac{1}{4}(1 - 3e^{-2})$ (c) $\frac{1}{2}(e^{-2} - 1)$
(d) $\frac{1}{4}(e^{-3} + 2e^{-2} - 1)$ (e) $\frac{1}{4}(1 - e^{-3} + 2e^{-2})$

8.(6 pts.) Evaluate

$$\int \frac{x^2}{\sqrt{9 - x^2}} dx.$$

- (a) $\frac{1}{2}x\sqrt{9 - x^2} + C$ (b) $\frac{9}{2} \left[\arcsin(x/3) - \frac{x}{3} \right] + C$
(c) $\frac{9}{2} \left[\arcsin(x/3) - \frac{x\sqrt{9 - x^2}}{9} \right] + C$ (d) $\frac{9}{2} \left[\arcsin(x/3) - \frac{x^2}{9} \right] + C$
(e) $9 \arcsin(x/3) + C$

9.(6 pts.) Evaluate

$$\int \frac{x+5}{x^2+x-2} dx.$$

(a) $\ln|x^2+x-2|+C$

(b) $\ln\left|\frac{2(x+2)}{x-1}\right|+C$

(c) $\ln(2|x-1|-|x+2|)+C$

(d) $\ln\left|\frac{(x-1)^2}{x+2}\right|+C$

(e) $\ln\left|\frac{(x+2)^2}{x-1}\right|+C$

10.(6 pts.) Find the Midpoint Rule approximation (using four intervals) of

$$\int_0^4 x^2 dx .$$

(a) 22

(b) 64/3

(c) $\frac{119}{4}$

(d) 21

(e) $\frac{95}{4}$

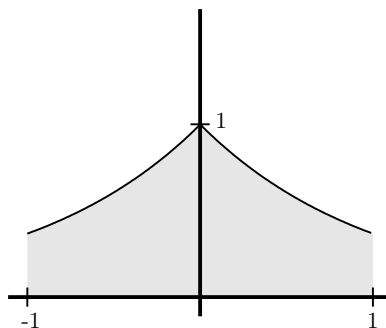
11.(6 pts.) Find the arclength of $y = 4 - 2x^{3/2}$, for $0 \leq x \leq 2$?

- (a) $\frac{1}{9}[\sqrt{19} - 1]$ (b) $\frac{2}{3}[19\sqrt{19}]$ (c) $\frac{20}{3}\sqrt{10}$
 (d) $\frac{2}{27}[10\sqrt{10} - 1]$ (e) $\frac{2}{27}[19\sqrt{19} - 1]$

12.(6 pts.) Find the center of mass of a plate with shape bounded below by the x -axis, on the left by $x = -1$, on the right by $x = 1$, and above by the curve

$$y = \begin{cases} e^x & -1 \leq x \leq 0 \\ e^{-x} & 0 \leq x \leq 1 \end{cases}.$$

The area is $2(1 - e^{-1})$.



- (a) $\left(0, \frac{e^{-2} - 2}{8(1 - e^{-1})}\right)$ (b) $(0, 2(1 - e^{-1}))$ (c) $\left(0, \frac{2(2 - e^{-2})}{1 - e^{-1}}\right)$
 (d) $\left(0, \frac{1 - e^{-2}}{4}\right)$ (e) $\left(0, \frac{1 - e^{-2}}{4(1 - e^{-1})}\right)$

13.(6 pts.) Solve the initial value problem:

$$y' = (3 - 2x)(1 + y) \quad y(1) = 0.$$

(a) $y(x) = e^{x^2-3x} - e^{-2}$ (b) $y(x) = e^{-2+3x-x^2} - 1$ (c) $y(x) = e^{3x-x^2} - e^2$

(d) $y(x) = e^{-2+3x-x^2}$ (e) $y(x) = e^{-1+3x-x^2} - e$

14.(6 pts.) If 100 grams of radioactive material with a half-life of two days are present at day zero, how many grams are left at day three?

(a) $\frac{100}{\sqrt{8}}$ (b) $\frac{100}{\sqrt{2}}$ (c) $\frac{100}{2^{1/3}}$ (d) $\frac{100}{4^{1/3}}$ (e) 50

15.(6 pts.) If the function $y(x)$ satisfies the differential equation

$$xy' - 2y = x - 2$$

subject to the initial value $y(1) = 1$, what is $y(2)$?

- (a) 5 (b) 4 (c) 3 (d) 2 (e) 1

16.(6 pts.) Which integral below computes the length of the parameterized curve $x(t) = t^3 + t$ and $y(t) = t^5 - t$ for $0 \leq t \leq 1$.

- (a) $\int_0^1 (t^3 + t)\sqrt{2 + 6t^2 - t^4 + 25t^8} dt$ (b) $\int_0^1 (t^5 - t)\sqrt{t^3 + t^5} dt$
(c) $\int_0^1 \sqrt{t^3 + t^5} dt$ (d) $\int_0^1 \sqrt{2 + 6t^2 - t^4 + 25t^8} dt$
(e) $\int_0^1 (t^5 - t)(3t^2 + 1) dt$

17.(6 pts.) Which integral below gives the area inside the polar curve $r = \sin(3\theta)$?

(a) $\frac{1}{2} \int_0^\pi \sin^2(3\theta) d\theta$

(b) $\frac{1}{2} \int_{\pi/6}^{\pi/3} \sin^2(3\theta) d\theta$

(c) $\frac{1}{2} \int_0^\pi \sqrt{\sin^2(3\theta) + 9 \cos^2(3\theta)} d\theta$

(d) $\frac{1}{2} \int_0^{2\pi} \sin^2(3\theta) d\theta$

(e) $\frac{1}{2} \int_0^{2\pi} \sqrt{\sin^2(3\theta) + 9 \cos^2(3\theta)} d\theta$

18.(6 pts.) Which statement below is true about the series $\sum_{n=1}^{\infty} \frac{2^n}{n^3 + 2^n}$

(a) $\lim_{n \rightarrow \infty} \frac{2^n}{n^3 + 2^n} = 0$ so the series converges.

(b) $\lim_{n \rightarrow \infty} \frac{2^n}{n^3 + 2^n} = 1$ so the series converges.

(c) $\lim_{n \rightarrow \infty} \frac{2^n}{n^3 + 2^n} = 1$ so the series diverges.

(d) $\lim_{n \rightarrow \infty} \frac{2^n}{n^3 + 2^n}$ does not exist so the series converges.

(e) $\lim_{n \rightarrow \infty} \frac{2^n}{n^3 + 2^n} = 0$ so the series diverges.

19.(6 pts.) Sum the series $\sum_{n=1}^{\infty} \frac{3^n}{5^{2n}}$.

(a) $\frac{25}{22}$

(b) $\frac{3}{22}$

(c) $\frac{5}{3}$

(d) $\frac{96}{25}$

(e) $\frac{50}{11}$

20.(6 pts.) The interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{\sqrt{n}}$$

is

(a) $[2, 4]$

(b) $(-1, 1)$

(c) $(-4, -2)$

(d) $(2, 4)$

(e) $[-4, -2)$

21.(6 pts.) Which series below absolutely converges?

- (a) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)}$ (c) $\sum_{n=1}^{\infty} \frac{\sqrt{n^3}}{n^2+1}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \pi^n}{3^n}$
- (e) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$

22.(6 pts.) Which series below conditionally converges? **These are the same series as in problem 21.**

- (a) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)}$ (c) $\sum_{n=1}^{\infty} \frac{\sqrt{n^3}}{n^2+1}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \pi^n}{3^n}$
- (e) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$

23.(6 pts.) Find a power series for the function

$$f(x) = -\frac{3}{(x+2)^2}.$$

$$(a) \quad \frac{3}{2} \sum_{n=1}^{\infty} (-1)^n n \frac{x^{n-1}}{2^n} \quad (b) \quad \frac{3}{2} \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1) 2^n} \quad (c) \quad \frac{3}{2} \sum_{n=1}^{\infty} n \frac{x^{n-1}}{2^n}$$

$$(d) \quad \frac{3}{2} \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{2^n} \quad (e) \quad \frac{3}{2} \sum_{n=1}^{\infty} (-1)^n n \frac{x^n}{2^n}$$

24.(6 pts.) Which series below represents $\frac{\sin x}{x}$?

$$(a) \quad \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \quad (b) \quad \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!} \quad (c) \quad \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$$

$$(d) \quad \sum_{n=0}^{\infty} (-1)^n \binom{1/2}{n} x^{2n} \quad (e) \quad \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$$

25.(6 pts.) Which series below gives the arc length of the curve $y = \frac{x^2}{2}$ from $x = 0$ to $x = 1$?

(a) $\sum_{n=0}^{\infty} \binom{1/2}{n} \frac{1}{2n+2}$ (b) $\sum_{n=0}^{\infty} \binom{1/3}{n} \frac{1}{2n+1}$ (c) $\sum_{n=0}^{\infty} \binom{1/3}{n} \frac{1}{2n+2}$

(d) $\sum_{n=0}^{\infty} \binom{1/2}{n} \frac{(-1)^n}{2n+1}$ (e) $\sum_{n=0}^{\infty} \binom{1/2}{n} \frac{1}{2n+1}$

Math 126
Final Exam
May 6, 2004

Name: _____

Instructor: _____

- Be sure that you have all 14 pages of test version I.
- No calculators are to be used.
- The exam lasts for two hours.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Please mark your answers with an **X!** Do NOT circle them!

The dotted lines in the answer box indicate page breaks.

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| 1. (a) (b) (c) (d) (e) | 15. (a) (b) (c) (d) (e) |
| 2. (a) (b) (c) (d) (e) | 16. (a) (b) (c) (d) (e) |
| | |
| 3. (a) (b) (c) (d) (e) | 17. (a) (b) (c) (d) (e) |
| 4. (a) (b) (c) (d) (e) | 18. (a) (b) (c) (d) (e) |
| | |
| 5. (a) (b) (c) (d) (e) | 19. (a) (b) (c) (d) (e) |
| 6. (a) (b) (c) (d) (e) | 20. (a) (b) (c) (d) (e) |
| | |
| 7. (a) (b) (c) (d) (e) | 21. (a) (b) (c) (d) (e) |
| 8. (a) (b) (c) (d) (e) | 22. (a) (b) (c) (d) (e) |
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| 9. (a) (b) (c) (d) (e) | 23. (a) (b) (c) (d) (e) |
| 10. (a) (b) (c) (d) (e) | 24. (a) (b) (c) (d) (e) |
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| 11. (a) (b) (c) (d) (e) | 25. (a) (b) (c) (d) (e) |
| 12. (a) (b) (c) (d) (e) | Final Exam: _____ |
| | Previous Total: _____ |
| 13. (a) (b) (c) (d) (e) | Course Total: _____ |
| 14. (a) (b) (c) (d) (e) | |

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Final Exam
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Instructor: ANSWER

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