Multiple Choice

1.(5 pts.)   Solve the following equation for x:

$$\ln(x+1) - \ln(x+3) = e .$$

(a)   There is no solution. (b) $\dfrac{1 - e^e}{3e^e - 1}$ (c) $\dfrac{3e^e - 1}{1 - e^e}$

(d) $x = -1$ and $x = 3$ (e) $x = 1$ and $x = -3$

2.(5 pts.)   Find the derivative of $(x^2 + 1)^x$.

(a) $(x^2 + 1)^x \ln(x^2 + 1)$ (b) $(2x)^x + x \ln(x^2 + 1)$

(c) $x(x^2 + 1)^{x-1}$ (d) $(x^2 + 1)^x \left(\ln(x^2 + 1) + \dfrac{2x^2}{x^2 + 1} \right)$

(e)   This function is not defined and hence has no derivative.

3.(5 pts.)   Evaluate $\displaystyle\int_0^{\pi/3} \tan\theta \, d\theta$.

(a) $\ln 2$ (b) $\ln 2 - \ln 3$ (c) $\ln 3 - \ln 2$ (d) $\ln 3$ (e) $\ln(1/2)$

4.(5 pts.)   Compute $\displaystyle\int_0^{\pi/2} (\cos x) \, 2^{\sin x} \, dx$.

(a) $\dfrac{\ln 2}{2}$ (b) $\dfrac{1}{\ln 2}$ (c) $\dfrac{2}{\ln 2}$ (d) $-\dfrac{1}{\ln 2}$ (e) $\dfrac{1}{\ln \sqrt{2}}$

5.(5 pts.)   Find $\displaystyle\lim_{x \to \infty} \dfrac{2^x}{x^5}$.

(a) $\dfrac{1}{32}$ (b) 32 (c) $-\infty$ (d) $+\infty$

(e)   Does not exist but is not $\pm\infty$.

6.(5 pts.) Evaluate $\int_3^5 \dfrac{1}{1+x^2}\,dx$

(a) $\ln(26) - \ln(10)$ (b) $\cot(26) - \cot(10)$ (c) $\ln(5) - \ln(3)$

(d) $\cot(5) - \cot(3)$ (e) $\arctan(5) - \arctan(3)$

7.(5 pts.) Find the derivative of $\arcsin(\sqrt{x}\,)$.

(a) $\dfrac{\arccos(\sqrt{x}\,)}{2\sqrt{x}}$ (b) $\dfrac{-1}{2\sqrt{x}\,\sqrt{1-x}}$ (c) $\dfrac{1}{2\sqrt{x}\,\sqrt{1-x}}$ (d) $\dfrac{1}{\sqrt{1-x}}$

(e) $\dfrac{-1}{\sqrt{1-x}}$

8.(5 pts.) Find $\lim\limits_{t\to 0}\dfrac{\arcsin(t)}{t}$.

(a) 1 (b) $+\infty$ (c) $\dfrac{\pi}{6}$ (d) 0

(e) Does not exist and is not $+\infty$

9.(5 pts.) Evaluate $\int \sin^2 x\,dx$.

(a) $\dfrac{x^2}{3} + \dfrac{\arcsin(x)\,\sin^2 x}{4} + C$ (b) $\dfrac{\cos^3(x)}{3} + C$

(c) $\dfrac{\sin^3 x}{3\cos x} + C$ (d) $\dfrac{x}{2} - \dfrac{\sin x\,\cos x}{2} + C$

(e) 0 because $\sin^2 x$ is an even function

10.(5 pts.) Find the slope of the tangent line to the curve $y = x + \arctan(y)$ at the point $\left(\dfrac{\pi}{4} - 1, \dfrac{\pi}{4}\right)$.

(a) $\dfrac{\pi}{6}$ (b) $1 + \dfrac{16}{\pi^2}$ (c) $\dfrac{\pi}{3} + \dfrac{4}{\pi}$ (d) $\dfrac{\pi}{3}$ (e) $\dfrac{\pi}{2} + \dfrac{16}{\pi^2}$

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11. (5 pts.) Find $f'(x)$ if $f(x) = \dfrac{(x+1)^3(x^2-3x+1)^5}{\sin^6 x}$.

(a) $\left(\dfrac{(x+1)^3(x^2-3x+1)^5}{\sin^6 x} \right) \left(\dfrac{3(x+1) + 5(x^2-3x+1)(2x-3) - 6\sin^5 \cos x}{\sin^6 x} \right)$

(b) $\left(\dfrac{(x+1)^3(x^2-3x+1)^5}{\sin^6 x} \right) \left(3 \cdot \dfrac{1}{x+1} - 5 \cdot \dfrac{2x-3}{x^2-3x+1} + 6 \cdot \dfrac{\cos x}{\sin x} \right)$

(c) $\left(\dfrac{(x+1)^3(x^2-3x+1)^5}{\sin^6 x} \right) \left(\dfrac{3(x+1) + 5(x^2-3x+1)(2x-3) + 6\sin^5 \cos x}{\sin^6 x} \right)$

(d) $\left(\dfrac{(x+1)^3(x^2-3x+1)^5}{\sin^6 x} \right) \left(\dfrac{15 \cdot \dfrac{2x-3}{(x+1)(x^2-3x+1)}}{6 \cdot \dfrac{\cos x}{\sin x}} \right)$

(e) $\left(\dfrac{(x+1)^3(x^2-3x+1)^5}{\sin^6 x} \right) \left(3 \cdot \dfrac{1}{x+1} + 5 \cdot \dfrac{2x-3}{x^2-3x+1} - 6 \cdot \dfrac{\cos x}{\sin x} \right)$

Partial Credit
You must show your work on the partial credit problems to receive credit!

12. (15 pts.)
A. Show that the function $f(x) = 2x + \sin(x)$ has an inverse function on the interval $(-\infty, \infty)$.
B. Compute the derivative of the inverse function at 2π.

13. (15 pts.) Evaluate $\displaystyle \int \dfrac{e^{\frac{1}{x}}}{x^3}\, dx$.

14. (15 pts.) Evaluate $\displaystyle \lim_{x\to 0} \dfrac{\ln(1+x) - x}{x \sin x}$.

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Instructor: ANSWERS

Exam I

February 4, 2003

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 4 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.   ($\bullet$)   (b)   (c)   (d)   (e)

2.   (a)   (b)   (c)   ($\bullet$)   (e)

3.   ($\bullet$)   (b)   (c)   (d)   (e)

4.   (a)   ($\bullet$)   (c)   (d)   (e)

5.   (a)   (b)   (c)   ($\bullet$)   (e)

6.   (a)   (b)   (c)   (d)   ($\bullet$)

7.   (a)   (b)   ($\bullet$)   (d)   (e)

8.   ($\bullet$)   (b)   (c)   (d)   (e)

9.   (a)   (b)   (c)   ($\bullet$)   (e)

10.   (a)   ($\bullet$)   (c)   (d)   (e)

11.   (a)   (b)   (c)   (d)   ($\bullet$)

DO NOT WRITE IN THIS BOX!

Total multiple choice: _____

12. _____

13. _____

14. _____

Total: _____