

Exam I
February 3, 2004

12. Use logarithmic differentiation to find $f'(x)$ if

$$f(x) = \frac{x \sin^2(x+7)}{(2x^3+2)^8}.$$

$$\ln(f(x)) = \ln(x) + 2 \ln(\sin(x+7)) - 8 \ln(2x^3+2)$$

so

$$\frac{f'(x)}{f(x)} = \frac{1}{x} + \frac{2 \cos(x+7)}{\sin(x+7)} - \frac{8(6x^2)}{2x^3+2}$$

and

$$f'(x) = \left(\frac{x \sin^2(x+7)}{(2x^3+2)^8} \right) \left(\frac{1}{x} + \frac{2 \cos(x+7)}{\sin(x+7)} - \frac{48x^2}{2x^3+2} \right)$$

13. Evaluate the following limit.

$$\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x}.$$

Use l'Hospital's Rule twice:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x} &= \lim_{x \rightarrow \infty} \frac{2(\ln x)(1/x)}{1} \quad [\text{l'Hospital for } \infty/\infty] \\ &= \lim_{x \rightarrow \infty} \frac{2 \ln x}{x} \\ &= \lim_{x \rightarrow \infty} \frac{2(1/x)}{1} \quad [\text{l'Hospital for } \infty/\infty] \\ &= \lim_{x \rightarrow \infty} \frac{2}{x} \\ &= 0. \end{aligned}$$

14. To evaluate $\int x \ln x \, dx$ apply integration by parts. There are several ways to proceed.

Choice 1: $dv = dx$. Then $u = x \ln x$; $v = x$ and $du = (\ln x + x \frac{1}{x}) dx = (\ln x + 1) dx$, so

$$\int x \ln x \, dx = x^2 \ln x - \int x(\ln x + 1) dx = x^2 \ln x - \int x \ln x \, dx - \int x \, dx = x^2 \ln x - \int x \ln x \, dx - \frac{1}{2} x^2.$$

so

$$\int x \ln x \, dx = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

Choice 2: $dv = x \, dx$. Then $u = \ln x$; $v = \frac{1}{2} x^2$ and $du = \frac{1}{x} \, dx$, so $\int x \ln x \, dx = \frac{1}{2} x^2 \ln x -$

$$\int \frac{1}{2} x^2 \frac{1}{x} \, dx = \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x \, dx = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C.$$

Choice 3: $dv = \ln x \, dx$. Then $u = x$; $v = x \ln x - x$ and $du = dx$, so $\int x \ln x \, dx =$

$$x^2 \ln x - x^2 - \int (x \ln x - x) dx = x^2 \ln x - x^2 - \int x \ln x \, dx - \int x \, dx = x^2 \ln x - x^2 + \int x \ln x \, dx + \frac{1}{2} x^2 = x^2 \ln x - \frac{1}{2} x^2 + \int x \ln x \, dx.$$

Solve for the integral as in Choice 1.
