**13.** Note that you may divide numerator and denominator by $x$ first if you wish. If you do not divide by $x$ first, the form is

$$\frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2}$$

If you do divide by $x$ first, then leave off the first term.

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**14.** Since $b - a = 4$, and $n = 8$, we have $\frac{\Delta x}{3} = \frac{1}{6}$. The approximation is therefore

$$\frac{1}{6}\left[f(0) + 4f\left(\frac{1}{2}\right) + 2f(1) + 4f\left(\frac{3}{2}\right) + 2f(2) + 4f\left(\frac{5}{2}\right) + 2f(3) + 4f\left(\frac{7}{2}\right) + f(4)\right]$$

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**15.** We can use the technique of partial fractions to replace the given fraction by a sum of terms which we hope will be easier to integrate, so

$$\frac{2x^2}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

or

$$(*) \qquad 2x^2 = A(x^2+1) + (Bx+C)(x+1)\ .$$

One can equate coefficients and get the system

$$\begin{array}{cc} x^2 : & A + B = 2 \\ x : & B + C = 0 \\ constant : & A + C = 0 \end{array}$$

From the last two equations, $A = B$ and hence $2A = 2$: $A = 1$ and then $B = 1$ and $C = -1$.

OR, plug in $x = -1$ into $(*)$ to get $2(-1)^2 = A(2)$, or $A = 1$ and then plug in $x = 0$ to get $0 = 1 + C$ so $C = -1$ and then plug in $x = 1$ to get $2 = 1(2) + (B-1)(2)$ or $B = 1$.

One can also do a judicious mixture of the two techniques.

In any case, $\displaystyle\int_0^1 \frac{2x^2}{(x+1)(x^2+1)}\,dx = \int_0^1 \frac{dx}{x+1} + \int_0^1 \frac{x-1}{x^2+1}\,dx.$

$$\int_0^1 \frac{dx}{x+1} = \ln|x+1|\,\Big|_0^1 = \ln(2) - \ln(1) = \ln(2).$$

$\displaystyle\int_0^1 \frac{x-1}{x^2+1}\,dx = \int_0^1 \frac{x}{x^2+1}\,dx - \int_0^1 \frac{dx}{x^2+1} = \frac{1}{2}\ln|x^2+1| - \arctan(x)\Big|_0^1 = \frac{1}{2}\ln(2) -$ $\arctan(1) - \big(\ln(1) - \arctan(0)\big)$. Since $\arctan(1) = \frac{\pi}{4}$ and $\arctan(0) = 0$, $\displaystyle\int_0^1 \frac{x-1}{x^2+1}\,dx =$ $\frac{1}{2}\ln(2) - \frac{\pi}{4}$ and so $\displaystyle\int_0^1 \frac{2x^2}{(x+1)(x^2+1)}\,dx = \frac{3}{2}\ln(2) - \frac{\pi}{4}$.

You can also do $\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx$ by substitution: $u = x^2+1$, $du = 2x\,dx$, so $\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx = \frac{1}{2}\int_1^2 \frac{du}{u} = \ln|u|\,\Big|_1^2 = \frac{1}{2}\big(\ln(2) - \ln(0)\big) = \frac{1}{2}\ln 2$.

If you forgot $\displaystyle\int_0^1 \frac{dx}{x^2+1}$ you can do a trig. substitution: $x = \tan\theta$, $dx = \sec^2\theta\,d\theta$, so $\displaystyle\int_0^1 \frac{dx}{x^2+1} = \int_0^{\pi/4} \frac{\sec^2\theta\,d\theta}{\sec^2\theta} = \int_0^{\pi/4} d\theta = \frac{\pi}{4}$.

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**15.** Divide through by x and rearrange terms in the equation as follows:

$$\frac{dy}{dx} + y(1 + \frac{1}{x}) = 1$$

Now in standard notation we have that $P(x) = 1 + \frac{1}{x}$ and $Q(x) = 1$. This yields

$$\int P(x)dx = \int (1 + \frac{1}{x})dx = x + \ln x$$

and

$$I(x) = e^{\int P(x)dx} = e^{(x+\ln x)} = e^x * e^{\ln x} = xe^x \ .$$

The formula for a solution of the equation has the following form

$$y(x) = \frac{1}{I(x)}(\int I(x)Q(x)\,dx + C) = \frac{1}{xe^x}\int xe^x \cdot 1\,dx + \frac{C}{xe^x} \ .$$

Integrate by parts: $u = x$, $dv = e^x dx$. Then $du = dx$ and $v = e^x$ so

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C \ .$$

The solution is

$$y(x) = \frac{1}{xe^x}(xe^x - e^x + C) = 1 - \frac{1}{x} + \frac{C}{xe^x} \ .$$

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