Multiple Choice

1.(5 pts.) Determine which curve is described by the following equation

$$r = 5\sin\theta.$$

(a) A hyperbola with foci on the y axis.

(b) A non–circular ellipse with foci on the y axis.

(c) A circle with center on the y axis.

(d) A parabola with directrix perpendicular to the y axis.

(e) A non–circular ellipse with center on the x axis.

2.(5 pts.) Find the length of a curve with polar equation: $r = 3^{(\theta+1)}, \ 0 \le \theta \le 2\pi$.

(a) $(3^{(2\pi+1)} - 3)$

(b) $\dfrac{\sqrt{1+3^2}(3^{(2\pi+1)} - 3)}{\ln 3}$

(c) $\dfrac{\sqrt{1+(\ln 3)^2}(3^{(2\pi+1)} - 3)}{\ln 3}$

(d) $\dfrac{\sqrt{(\ln 3)^2 - 1}(3^{(2\pi+1)} - 1)}{\ln 3}$

(e) $3(3^{(2\pi+1)} - 1)$

3.(5 pts.) Find the foci and asymptotes of the hyperbola $16x^2 - 9y^2 = 16$.

(a) foci are points $\left(\pm\frac{3}{2}, 0\right)$ and asymptotes are the lines $y = \frac{x}{2}$ and $y = -\frac{x}{2}$

(b) foci are points $\left(\pm\frac{5}{3}, 0\right)$ and asymptotes are the lines $y = \frac{x}{3}$ and $y = -\frac{x}{3}$

(c) foci are points $\left(\pm\frac{3}{5}, 0\right)$ and asymptotes are the lines $y = \frac{4x}{3}$ and $y = -\frac{4x}{3}$

(d) foci are points $\left(\pm\frac{1}{3}, 0\right)$ and asymptotes are the lines $y = \frac{5x}{3}$ and $y = -\frac{5x}{3}$

(e) foci are points $\left(\pm\frac{5}{3}, 0\right)$ and asymptotes are the lines $y = \frac{4x}{3}$ and $y = -\frac{4x}{3}$

4.(5 pts.) Find an equation for the ellipse with foci $(0, \pm 2)$ and vertices $(0, \pm 3)$.

(a) $\frac{x^2}{5} + \frac{y^2}{9} = 1$ (b) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ (c) $\frac{(x-2)^2}{4} + \frac{y^2}{9} = 1$

(d) $\frac{x^2}{9} + \frac{y^2}{4} = 1$ (e) $\frac{x^2}{9} + \frac{y^2}{5} = 1$

5.(5 pts.) Which of the following series are convergent?

$$1) \sum_{k=1}^{\infty} \frac{1}{2^k} \; ; \qquad 2) \sum_{k=1}^{\infty} \frac{1}{k} \; ; \qquad 3) \sum_{k=1}^{\infty} \frac{2}{\sqrt{k}} \; ; \qquad 4) \sum_{k=1}^{\infty} \frac{1}{(k-1)^2} \; ; \qquad 5) \sum_{k=1}^{\infty} \frac{1}{(4k)^2} \; .$$

(a) 1), 2), 5) (b) 3), 4) (c) 1), 4), 5) (d) 1), 3), 4), 5) (e) 1), 5)

6.(5 pts.) Determine which of the following series are convergent.

$$1) \sum_{k=1}^{\infty} \frac{1}{k!} \; ; \qquad 2) \sum_{k=1}^{\infty} \frac{1}{ke^{-k}} \; ; \qquad 3) \sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}$$

(a) 1), 2), 3) (b) 1), 2) (c) 2), 3) (d) 1), 3) (e) 3)

7.(5 pts.) Find a power series representation for $\int_0^x \ln(1-t)\, dt$.

(a) $\sum_{n=1}^{\infty} \frac{-x^n}{n(n+1)}$ (b) $\sum_{n=1}^{\infty} \frac{x^n}{2^n}$

(c) $\sum_{n=1}^{\infty} \frac{x^n}{n}$ (d) $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{n^2-1}$

(e) $\sum_{n=1}^{\infty} \frac{-x^n}{n+1}$

8.(5 pts.) Find the interval of convergence of the following power series

$$\sum_{k=1}^{\infty} \frac{x^k}{2^k}.$$

(a) $[-2, 2)$ (b) $(-2, 2]$ (c) $(-1, 1]$ (d) $(-2, 2)$ (e) $[-1, 1]$

9.(5 pts.) Consider the polar equation $r = \dfrac{12}{1 - 2\cos\theta}$. Which of the following statements is correct?

(a) The graph is an ellipse with vertices 8 units apart.

(b) The graph is a conic section with distance from the center to the focus being 6 units.

(c) The graph is an ellipse with directrix $x = -6$.

(d) The graph is a hyperbola with directrix $y = -6$.

(e) The graph is a hyperbola with vertices 8 units apart.

10.(5 pts.) For each of the following series determine whether it diverges or converges absolutely or conditionally.

$$1) \sum_{k=0}^{\infty} (-1)^k \frac{2}{k!} \; ; \qquad 2) \sum_{k=0}^{\infty} (-1)^{(k+1)} \frac{k}{k^2+1} \; ; \qquad 3) \sum_{k=0}^{\infty} (-1)^{(k+1)} \frac{k}{2k+1} \; .$$

(a) 1) conditionally convergent 2) conditionally convergent 3) absolutely convergent.

(b) 1) absolutely convergent 2) conditionally convergent 3) divergent.

(c) 1) absolutely convergent 2) absolutely convergent 3) divergent.

(d) 1) absolutely convergent 2) conditionally convergent 3) conditionally convergent.

(e) 1) divergent 2) conditionally convergent 3) conditionally convergent.

11.(5 pts.) Find a power series representation for $f(x) = \dfrac{5}{(1-x)^2}$ and determine its interval of convergence. Hint: You can obtain it by differentiating power series representation for $g(x) = \dfrac{5}{(1-x)}$.

(a) $\displaystyle\sum_{n=1}^{\infty} nx^n$, interval of convergence: $[-1, 1)$

(b) $\displaystyle\sum_{n=1}^{\infty} 5nx^{n-1}$, interval of convergence: $(-1, 1)$

(c) $\displaystyle\sum_{n=1}^{\infty} 5nx^{n-1}$, interval of convergence: $[-1, 1]$

(d) $\displaystyle\sum_{n=1}^{\infty} 5nx^{n-2}$, interval of convergence: $[-1, 1)$

(e) $\displaystyle\sum_{n=1}^{\infty} 10nx^{n-1}$, interval of convergence: $[-1, 1]$

Partial Credit

You must show your work on the partial credit problems to receive credit!

12.(5 pts.) Sum the following series:

$$\sum_{k=1}^{\infty} e^{2-k}.$$

13.(10 pts.) Find the radius of convergence and interval of convergence of the following power series

$$\sum_{k=1}^{\infty} \frac{x^k}{k}.$$

14.(15 pts.) Use the Comparison Test (not the Limit Comparison Test) to determine whether the following infinite series is convergent or divergent. Justify your answer.

$$\sum_{k=1}^{\infty} \frac{1}{2^k + k}.$$

15.(15 pts.) Find the area of the region that lies inside both curves: $r = 2\sqrt{3}\cos\theta$ and $r = 2\sin\theta$.

Name: ANSWERS

Instructor: ANSWERS

Exam III
April 17, 2003

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 70 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 6 pages of the test.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(•)	(d)	(e)
2.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(•)
4.	(•)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(•)	(d)	(e)
6.	(a)	(b)	(c)	(•)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(•)	(e)
9.	(a)	(b)	(c)	(d)	(•)
10.	(a)	(•)	(c)	(d)	(e)
11.	(a)	(•)	(c)	(d)	(e)

DO NOT WRITE IN THIS BOX!

Total multiple choice:	
12.	
13.	
14.	
15.	
Total:	