12. This is a geometric series with the first term being $e^{2-1} = e$, and the ratio of terms being $\dfrac{e^{2-(k+1)}}{e^{2-k}} = \dfrac{1}{e}$. The sum is therefore $\dfrac{e}{1-\dfrac{1}{e}} = \dfrac{e^2}{e-1}$.

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13. The ratio test may be used to determine the radius of convergence.

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\frac{|x|^{k+1}}{k+1}\frac{k}{|x|^k} = |x|$$

Hence the radius of convergence is one.

The left endpoint of the interval of convergence is -1, and $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k}{k}$ converges by the Alternating Series Test.

The right endpoint of the interval of convergence is $+1$, and $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k}$ diverges either because it is the harmonic series, or because it is a p-series with $p = 1$.

So the interval of convergence is $[-1, 1)$.

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14. To use the Comparison Test one has to find a series to which to compare the given one. Look at the kth term, $\dfrac{1}{2^k+k}$: this can easily be compared to

1) $\dfrac{1}{2^k+k} < \dfrac{1}{2^k}$
2) $\dfrac{1}{2^k+k} < \dfrac{1}{k}$

If you guessed 2), then $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k}$ is the harmonic series (or p–series with $p = 1$) and hence it diverges. But this says nothing about the original series.

Try 1): $\displaystyle\sum_{k=1}^{\infty}\frac{1}{2^k}$ is a geometric series with $r = \dfrac{1}{2} < 1$ and so it converges and now the Comparison Test says $\displaystyle\sum_{k=1}^{\infty}\frac{1}{2^k+k}$ converges.

It is also possible, although harder, to compare to $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^2}$. This is a p–series with $p = 2$ and so converges. Hence we would like to show $\dfrac{1}{2^k+k} < \dfrac{1}{k^2}$, at least for large k. Equivaelently, $k^2 < 2^k + k$. To do this, let $f(x) = 2^x + x - x^2$. As $x \to \infty$, $f \to \infty$ so $f(x) > 0$ for all large enough x. In other words, $x^2 < 2^x + x$ for all large x.

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15. Two curves have two points of intersection. First point is the origin: (0,0). In order to find second point we need to equate two expressions $2\sqrt{3}\cos\theta = 2\sin\theta$. This results in $\tan\theta = \sqrt{3}$. Lastly, we have $\theta = \frac{\pi}{3}$. The area of the domain bounded by two curves is given by the following formula

$$\frac{1}{2}\int_0^{\frac{\pi}{3}} (2\sin\theta)^2 \, d\theta + \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (2\sqrt{3}\cos\theta)^2 \, d\theta = 2\int_0^{\frac{\pi}{3}} \sin^2\theta \, d\theta + 6\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2\theta \, d\theta.$$

After using the following substitutions: $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$ and $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$ previous formula yields that

$$\int_0^{\frac{\pi}{3}} (1 - \cos 2\theta) \, d\theta + 3\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1 + \cos 2\theta) \, d\theta = \frac{5\pi}{6} - \sqrt{3}\,.$$
