Multiple Choice

1.(5 pts.) Find the solution to the differential equation with initial value:

$$x^2y' + xy = 3x^3 + 1 \qquad y(1) = 0$$

(a) $y = e^x(x^2 - 3x + 2)$ (b) $y = x^2 + \dfrac{\ln x}{x} - \dfrac{1}{x}$ (c) $y = \ln(x + e^x - 1) - 1.$

(d) $y = \dfrac{3x^4}{4} + x - \dfrac{7}{4}$ (e) $y = e^x - e$

2.(5 pts.) Which integral below computes the area above the x–axis and below the parameterized curve $x(t) = t - \sin(t) + e^t$ and $y(t) = \sin(t)$ for $0 \le t \le \pi$.

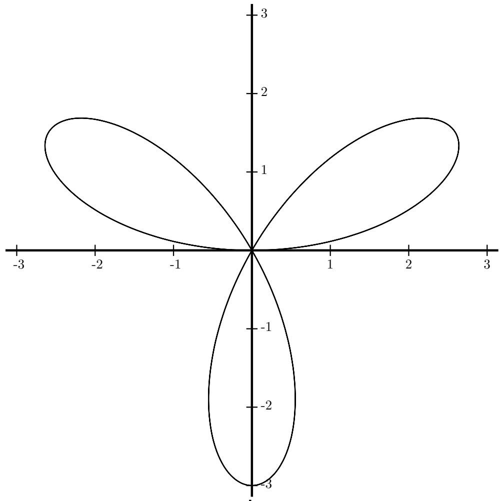
(a) $\displaystyle\int_0^{\pi} \cos(t)(1 - \cos(t) + e^t)\ dt$ (b) $\displaystyle\int_0^{\pi} \cos(t)(1 - \sin(t) + e^t)\ dt$

(c) $\displaystyle\int_0^{\pi} \sin(t)(1 + 2\cos(t) + e^t)\ dt$ (d) $\displaystyle\int_0^{\pi} \sin(t)(1 - \sin(t) + e^t)\ dt$

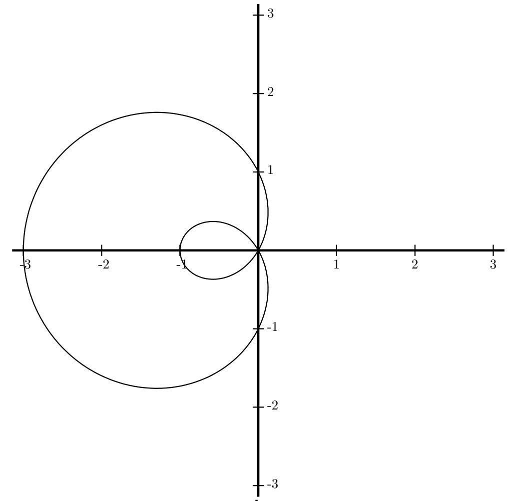
(e) $\displaystyle\int_0^{\pi} \sin(t)(1 - \cos(t) + e^t)\ dt$

3.(5 pts.) Which graph below is the graph of the polar equation $r = 3\sin(3\theta)$.

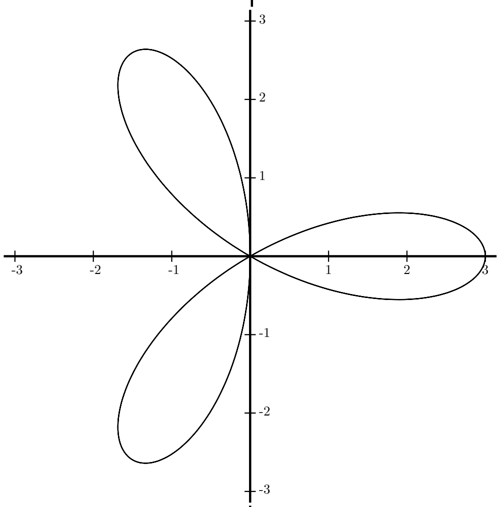
(a)



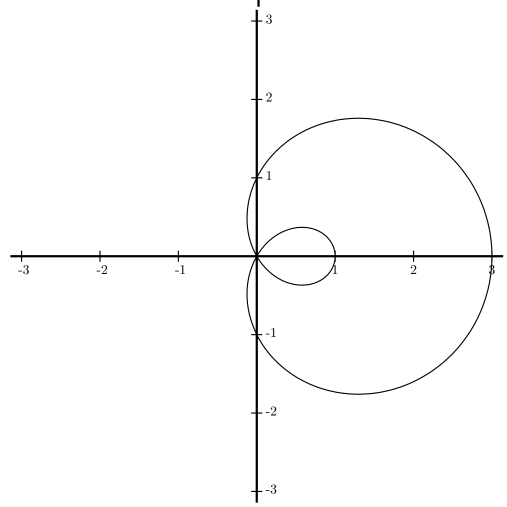
(b)



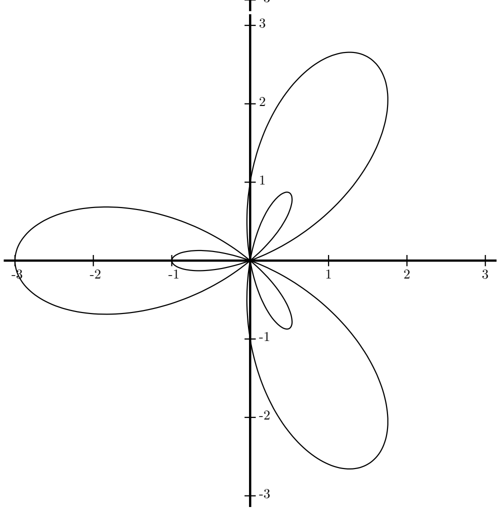
(c)



(d)



(e)



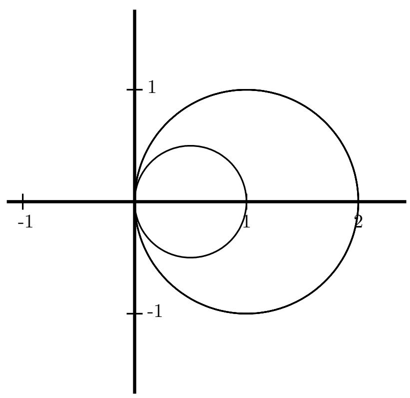
4.(5 pts.) Which integral below computes the length of the parameterized curve $x(t) = t + e^t$ and $y(t) = t^2 + e^t$ for $0 \le t \le \pi$.

(a) $\displaystyle\int_0^{\pi} \sqrt{t^2 + 2te^t + 2e^{2t} + t^4 + 2t^2e^t}\ dt$ (b) $\displaystyle\int_0^{\pi} \sqrt{t^2 + 6te^t + 2e^{2t} + 4t^2}\ dt$

(c) $\displaystyle\int_0^{\pi} \sqrt{1 + 2e^t + 2e^{2t} + t^4 + 2t^2e^t}\ dt$ (d) $\displaystyle\int_0^{\pi} \sqrt{1 + 2e^t + 2e^{2t} + 4t^2 + 4te^t}\ dt$

(e) $\displaystyle\int_0^{\pi} \sqrt{1 + 2e^t - 4t^2 - 4te^t}\ dt$

5.(5 pts.) Which integral below is the area of the region inside the circle $r = 2\cos\theta$ and outside the circle $r = \cos\theta$.



(a) $\displaystyle\frac{1}{2}\int_0^{\pi} 5\cos^2\theta\ d\theta$ (b) $\displaystyle\frac{1}{2}\int_0^{\pi} 3\cos^2\theta\ d\theta$ (c) $\displaystyle\frac{1}{2}\int_0^{\pi} 4\cos^2\theta\ d\theta$

(d) $\displaystyle\frac{1}{2}\int_0^{\pi} 3\sin^2\theta\ d\theta$ (e) $\displaystyle\frac{1}{2}\int_0^{\pi} 4\sin^2\theta\ d\theta$

6.(5 pts.) Find the length of the polar spiral $r = e^{\theta}$, $0 \le \theta \le 3$.

(a) $\sqrt{2}e^3 - 1$ (b) $e^3 - 1$ (c) $e^3 + 1$

(d) $\sqrt{2}\,(e^3 + 1)$ (e) $\sqrt{2}\,(e^3 - 1)$

7.(5 pts.) Which statement below is true about the series $\sum_{n=1}^{\infty} \frac{e^n}{n^2+e^n}$

(a) $\lim_{n\to\infty} \frac{e^n}{n^2+e^n}$ does not exist so the series converges.

(b) $\lim_{n\to\infty} \frac{e^n}{n^2+e^n} = 0$ so the series diverges.

(c) $\lim_{n\to\infty} \frac{e^n}{n^2+e^n} = 0$ so the series converges.

(d) $\lim_{n\to\infty} \frac{e^n}{n^2+e^n} = 1$ so the series diverges.

(e) $\lim_{n\to\infty} \frac{e^n}{n^2+e^n} = 1$ so the series converges.

8.(5 pts.) Sum the series $\sum_{n=2}^{\infty} \frac{2^n}{5^{2n}}$.

(a) $\frac{4}{23 \cdot 25}$ (b) $\frac{92}{25}$ (c) $\frac{5}{3}$ (d) $\frac{25}{23}$ (e) $\frac{100}{23}$

9.(5 pts.) Which of the statements below is true about the three series

I) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ II) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ III) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$

(a) I) diverges, II) conditionally converges and III) absolutely converges.

(b) They all conditionally converge.

(c) I) absolutely converges, II) conditionally converges and III) diverges.

(d) I) conditionally converges; II) and III) absolutely converge

(e) They all absolutely converge.

10.(5 pts.) Consider the following two series:

$$\text{I)} \sum_{n=2}^{\infty} \frac{1}{n^3 - 1} \qquad \text{and} \qquad \text{II)} \sum_{n=2}^{\infty} \frac{1}{n^3}\ .$$

Which statement below is true?

(a) Series II) converges but series I) diverges.

(b) None of the other statements are true.

(c) Both series converge.

(d) Both series diverge.

(e) Series I) converges but series II) diverges.

11.(5 pts.) Which series below conditionally converges?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} e^n}{\sqrt[5]{n^2}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[5]{n^4}}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[3]{n^4}}$

Partial Credit

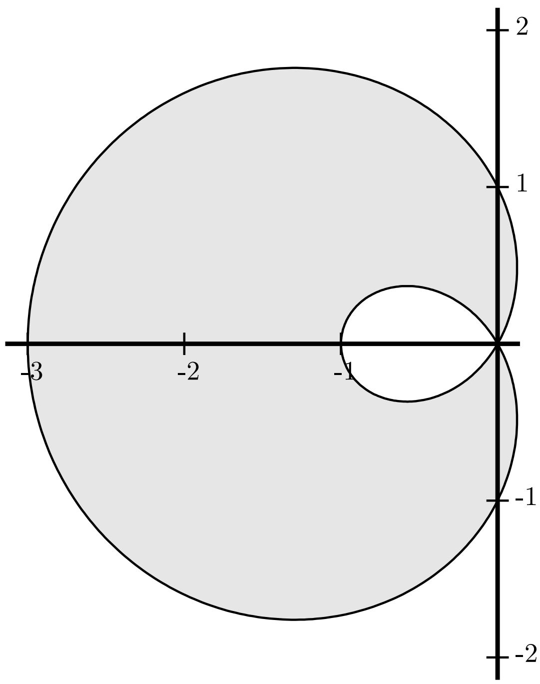
You must show your work on the partial credit problems to receive credit!

12.(15 pts.) Use the Integral Test to show that the series $\displaystyle\sum_{n=3}^{\infty} \frac{\ln n}{n}$ diverges.

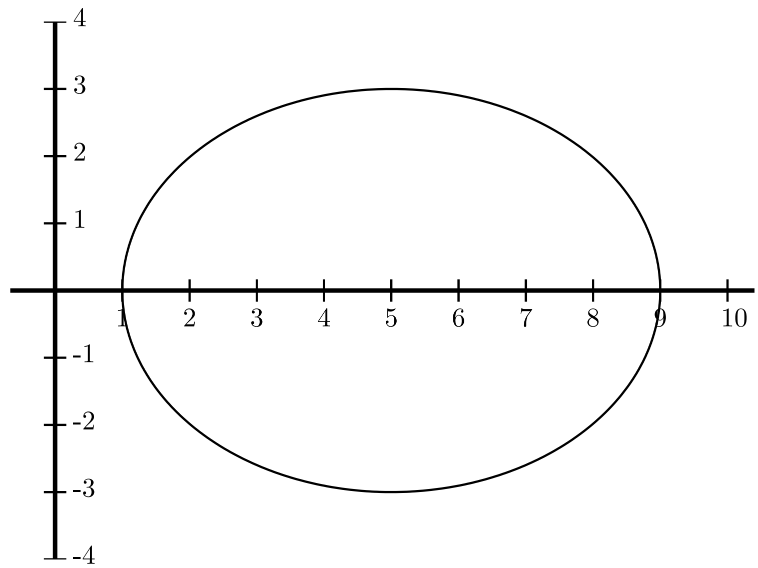
Remark: Be sure to check that the Integral Test can be applied.

13.(15 pts.) Set up an integral which computes the shaded area. The polar equation of the region is $r = 1 - 2\cos\theta$.

Remarks: $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$. The answer is the difference of two areas.



14.(15 pts.) Set up an integral which computes the surface area of the surface obtained by rotating the parameterized ellipse $x(t) = 5 + 4\cos(t)$, $y(t) = 3\sin(t)$, $0 \le t \le 2\pi$ about the y–axis .



Name: ANSWERS

Instructor: ANSWERS

Math 126 Exam III
April 20, 2004

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 8 pages of the test.
- The backs of pages may be used if you need additional room to work on a problem.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(•)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(•)
3.	(•)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(•)	(e)
5.	(a)	(•)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(•)
7.	(a)	(b)	(c)	(•)	(e)
8.	(•)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(•)	(e)
10.	(a)	(b)	(•)	(d)	(e)
11.	(a)	(b)	(•)	(d)	(e)

DO NOT WRITE IN THIS BOX!

Total multiple choice:	
12.	
13.	
14.	
Total:	