1.(6 pts.) Solve the following equation for x:

$$\ln(x+4) - \ln x = 1 \ .$$

(a) $x = \dfrac{4}{e-1}$ and $x = \dfrac{4}{e+1}$

(b) $x = e+2$ and $x = e-2$

(c) $x = \dfrac{4}{e-1}$

(d) There is no solution.

(e) $x = \dfrac{4}{1-e}$

2.(6 pts.) Find the derivative of $(x^2+1)^{x^2+1}$.

(a) $(x^2+1)^{x^2+1}\big(2x\ln(x^2+1)\big)$

(b) $2x(x^2+1)^{x^2}$

(c) $(x^2+1)^{x^2+1}\ 2x\big(\ln(x^2+1)+1\big)$

(d) $(x^2+1)^{x^2+1}$

(e) This function is not defined and hence has no derivative.

3.(6 pts.) Find $\lim\limits_{x\to\infty} \dfrac{3^x}{x^5-6x^4+3x^3+8x^2-117x+10}$.

(a) $\dfrac{1}{81}$

(b) $-\infty$

(c) $+\infty$

(d) 81

(e) Does not exist but is not ∞ or $-\infty$.

4.(6 pts.) Evaluate $\displaystyle\int_0^{\pi/3} \sec\theta \ d\theta$.

(a) Diverges

(b) $\ln\big(2+\sqrt{3}\ \big)$

(c) $\ln 3 - \ln 2$

(d) $\ln\big(1+\sqrt{3}\ \big)$

(e) $\ln 2 - \dfrac{\ln 3}{2}$

5.(6 pts.) $\displaystyle\int_0^{\frac{\pi}{2}} \sin^3 x \cos^2 x \ dx =$

(a) $\dfrac{2}{15}$

(b) $\dfrac{2}{45}$

(c) $\dfrac{3}{70}$

(d) $\dfrac{5}{12}$

(e) $\dfrac{4}{21}$

6.(6 pts.) In the partial fraction expansion of $\dfrac{x}{(x+1)(x+2)^2}$, the term with denominator $(x+2)$ has what numerator?

(a) $x+1$ (b) 1 (c) 3 (d) x (e) -2

7.(6 pts.) $\displaystyle\int_{-2}^{1} \frac{x^3+4x^2+13x+3}{x^2+4x+13}dx =$

(a) $\dfrac{1}{2}+\dfrac{\pi}{3}$ (b) $-\dfrac{3}{2}+\dfrac{\pi}{4}$ (c) $-1+\dfrac{3\pi}{4}$ (d) $\dfrac{5}{2}+\dfrac{3\pi}{4}$ (e) $-1+\dfrac{\pi}{6}$

8.(6 pts.) $\displaystyle\int_{1}^{\infty} \frac{dx}{x^2\sqrt{1+x^2}} =$

(a) $-1+\sqrt{2}$ (b) $-1+2\sqrt{2}$ (c) Divergent (d) 2 (e) $\sqrt{2}$

9.(6 pts.) $\displaystyle\int_{0}^{1} \frac{dx}{x^2-1} =$

(a) Divergent (b) $\ln\dfrac{\sqrt{3}+1}{2}$ (c) $2\ln 2$ (d) $\dfrac{\ln 2}{4}$ (e) $\dfrac{1}{\sqrt{3}}$

10.(6 pts.) Find the y-coordinate of the center of mass of the region bounded by $y=1-x^2$ and the x-axis. The area of this region is $\dfrac{4}{3}$.

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{5}$ (d) $\dfrac{3}{8}$ (e) $\dfrac{2}{5}$

11.(6 pts.) Sugar dissolves in water at a rate proportional to the amount still undissolved. If it takes 4 hours to reduce 30 pounds of sugar to 10 pounds, how many hours does it take to reduce 30 pounds of sugar to 3 pounds?

(a) 10.8 (b) $\dfrac{4\ln 10}{\ln 3}$ (c) $0.9(\ln 30-\ln 10)$ (d) $\dfrac{4\ln 20}{\ln 3}$

(e) 12

12.(6 pts.) If $x\dfrac{dy}{dx}+3y=\dfrac{4}{x}$, and $y(1)=10$, find $y(2)$.

(a) 0 (b) 7 (c) $\dfrac{4}{3}$ (d) 2 (e) $\dfrac{1}{2}$

13.(6 pts.) The curve given by $x = \cos^2 t, y = \sin t, 0 \leq t \leq \frac{\pi}{3}$, is rotated about the x-axis. The area of the resulting surface is given by which one of the following integrals?

(a) $2\pi \int_0^{\frac{\pi}{3}} \sin t \cos t \; dt$

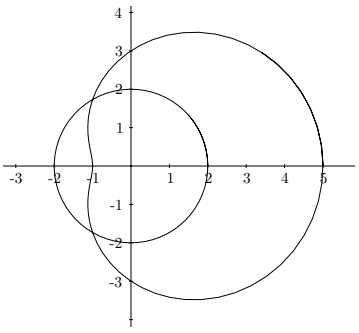
(b) $2\pi \int_0^{\frac{\pi}{3}} \sin t \sqrt{\cos^4 t + \sin^2 t} \; dt$

(c) $2\pi \int_0^{\frac{\pi}{3}} \cos^2 t \sqrt{\cos^4 t + \sin^2 t} \; dt$

(d) $2\pi \int_0^{\frac{\pi}{3}} \sin t \cos t \sqrt{4 \sin^2 t + 1} \; dt$

(e) $2\pi \int_0^{\frac{\pi}{3}} \sin t \sqrt{4 \sin^2 t + 1} \; dt$

14.(6 pts.) The area inside the curve $r = 3 + 2\cos\theta$ and outside the circle $r = 2$ is given by which integral below?



(a) $\int_0^{\frac{2\pi}{3}} \big(7 + 12\cos\theta + 2\cos(2\theta)\big) \, d\theta$

(b) $\int_0^{\frac{7\pi}{6}} \big(12 - 7\cos\theta + 2\cos(2\theta)\big) \, d\theta$

(c) $\int_0^{\frac{\pi}{3}} \big(7 + 12\cos\theta + 2\cos(2\theta)\big) \, d\theta$

(d) $\int_0^{\frac{2\pi}{3}} \big(12 - 7\cos\theta + 2\cos(2\theta)\big) \, d\theta$

(e) $\int_0^{\frac{\pi}{3}} \big(12 - 7\cos\theta + 2\cos(2\theta)\big) \, d\theta$

15.(6 pts.) Find the arc length of the curve with polar equation: $r = 2 - 2\cos\theta, \quad 0 \leq \theta \leq 2\pi$.

Hint: $\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2} - \frac{1}{2}\cos\theta$.

(a) 34 (b) 12 (c) 32 (d) 16 (e) 8

16.(6 pts.) Find the slope of the tangent line to the curve $r = 3\sin\theta$ at $\theta = 0$.

(a) 1 (b) -1 (c) 0 (d) π (e) 2

17.(6 pts.) Find an equation of the hyperbola with center at $(-2, 0)$, vertices at $(-4, 0)$ and $(0, 0)$ and foci at $(-5, 0)$ and $(1, 0)$.

(a) $\dfrac{(x+4)^2}{2} - \dfrac{y^2}{5} = 1$

(b) $\dfrac{(x-2)^2}{5} - \dfrac{y^2}{4} = 1$

(c) $\dfrac{(x+2)^2}{4} - \dfrac{y^2}{5} = 1$

(d) $\dfrac{(x-5)^2}{2} - \dfrac{y^2}{4} = 1$

(e) $\dfrac{(x+2)^2}{5} - \dfrac{y^2}{4} = 1$

18.(6 pts.) Determine the center and the foci of the ellipse

$$\frac{x^2}{9} + \frac{(y+1)^2}{25} = 1.$$

(a) center is at $(0, +1)$, the foci are at $(0, -5)$ and $(0, 3)$.

(b) center is at $(0, -1)$, the foci are at $(0, -4)$ and $(0, 2)$.

(c) center is at $(0, +1)$, the foci are at $(0, -4)$ and $(0, 2)$.

(d) center is at $(0, -1)$, the foci are at $(0, -5)$ and $(0, 3)$.

(e) center is at $(0, -1)$, the foci are at $(0, -6)$ and $(0, 4)$.

19.(6 pts.) Which of the following series are convergent and which are divergent?

$$1) \sum_{n=2}^{\infty} \frac{2}{n \ln n}; \qquad 2) \sum_{n=1}^{\infty} \frac{\sin n + 2}{n^2}; \qquad 3) \sum_{n=2}^{\infty} \frac{\ln n}{n}$$

(a) **1)** diverges **2)** diverges **3)** converges

(b) **1)** diverges **2)** diverges **3)** diverges

(c) **1)** diverges **2)** converges **3)** diverges

(d) **1)** converges **2)** converges **3)** converges

(e) **1)** converges **2)** converges **3)** diverges

20.(6 pts.) Determine if the series are absolutely convergent, conditionally convergent or divergent

$$\textbf{1)}\sum_{n=1}^{\infty}\frac{n^2}{e^n}; \qquad \textbf{2)}\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n}; \qquad \textbf{3)}\sum_{n=1}^{\infty}\frac{n!}{4^n}.$$

(a) **1)** absolutely convergent **2)** conditionally convergent **3)** absolutely convergent

(b) **1)** absolutely convergent **2)** absolutely convergent **3)** divergent

(c) **1)** divergent **2)** conditionally convergent **3)** conditionally convergent

(d) **1)** absolutely convergent **2)** conditionally convergent **3)** divergent

(e) **1)** divergent **2)** divergent **3)** conditionally convergent

21.(6 pts.) Determine intervals of convergence for the following power series

$$\textbf{1)}\sum_{n=0}^{\infty}\frac{2^n}{n!}(x-2)^n, \qquad \textbf{2)}\sum_{n=1}^{\infty}\frac{1}{n}(x-1)^n.$$

(a) **1)** $(-\infty,\infty)$ **2)** $(0,1)$ (b) **1)** $(0,2)$ **2)** $[-1,2)$

(c) **1)** $[-1,0)$ **2)** $[0,2)$ (d) **1)** $(-\infty,\infty)$ **2)** $[0,2)$

(e) **1)** $(-\infty,\infty)$ **2)** $(0,2)$

22.(6 pts.) Compute $\lim\limits_{x\to 0}\dfrac{\arctan x\ -\ \sin x}{e^{x^3}-1}$.

Hint: Use series.

(a) $\frac{\pi}{4}$ (b) $-\frac{1}{6}$ (c) $-\frac{5}{12}$ (d) $\frac{1}{3}$ (e) $\frac{4}{\pi}$

23.(6 pts.) What is the coefficient of x^3 in the Maclaurin series expansion of $(1+x)^{\frac{3}{4}}$?

(a) $\frac{15}{128}$ (b) $\frac{128}{27}$ (c) $\frac{1}{27}$ (d) 0 (e) $\frac{5}{128}$

24.(6 pts.) Which series below evaluates to the length of the curve $y = \dfrac{x^3}{3}$ from $x = 0$ to $x = 1$? **Hint:** First write an integral which gives the length. Then use power series methods to evaluate it.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{4n+1} \cdot \binom{\frac{1}{2}}{n}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{4n+1} \cdot \binom{\frac{1}{4}}{n}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{6n+1} \cdot \binom{\frac{1}{3}}{n}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n+1} \cdot \binom{\frac{1}{4}}{n}$ (e) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n+1} \cdot \binom{\frac{1}{2}}{n}$

25.(6 pts.) Find the coefficient of x^{4n+1} in the Maclaurin series for $x\cos(x^2)$.

(a) $\dfrac{(-1)^n}{(4n)!}$ (b) $\dfrac{(-1)^n}{(4n+1)!}$ (c) $\dfrac{1}{(4n)!}$ (d) $\dfrac{1}{(2n)!}$ (e) $\dfrac{(-1)^n}{(2n)!}$

Math 126
Final Exam
May 7, 2003

Name: _____

Instructor: ANSWER

- Be sure that you have all 7 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Please mark your answers with an **X**! Do NOT circle them!

The dotted lines in the answer box indicate page breaks.

1.	(a)	(b)	(•)	(d)	(e)	15.	(a)	(b)	(c)	(•)	(e)
2.	(a)	(b)	(•)	(d)	(e)	16.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(•)	(d)	(e)	17.	(a)	(b)	(•)	(d)	(e)
4.	(a)	(•)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(•)	(e)
5.	(•)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(•)	(d)	(e)
6.	(a)	(•)	(c)	(d)	(e)	20.	(a)	(b)	(c)	(•)	(e)
7.	(a)	(•)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(•)	(e)
8.	(•)	(b)	(c)	(d)	(e)	22.	(a)	(•)	(c)	(d)	(e)
9.	(•)	(b)	(c)	(d)	(e)	23.	(a)	(b)	(c)	(d)	(•)
10.	(a)	(b)	(c)	(d)	(•)	24.	(•)	(b)	(c)	(d)	(e)
11.	(a)	(•)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(•)
12.	(a)	(b)	(c)	(•)	(e)						
13.	(a)	(b)	(c)	(•)	(e)						
14.	(•)	(b)	(c)	(d)	(e)						

Final Exam: _____

Previous Total: _____

Course Total: _____