Math 126: Calculus II   Name:_____

Exam I   *October 3, 2002*   Section:_____

There are 6 problems on 6 pages (including the cover page) worth a total of 100 points. Unless otherwise indicated, each part of a problem is worth the same number of points.

You may use a calculator if you wish. However, all answers must be exact, e.g., 1.414 is not equal to $\sqrt{2}$.

To receive partial credit on a problem, you must *show your work* including *all important steps*. No credit will be given for an answer if no work is shown.

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

Total _____

1. (20 points) Consider the function $f(x) = \dfrac{e^{3x} - e^{-3x}}{2}$.

   (a) Write f in terms of hyperbolic trigonometric functions.

   (b) Show that it is increasing for all x.

   (c) What is its range?

   (d) What is the range of its inverse function $g = f^{-1}$?

   (e) Find $g'(a)$, where $a = 0$.

2. (20 points)

   (a) Find $\displaystyle\lim_{x\to0}\frac{\ln(1+2x)}{x}$.

   (b) Evaluate $\displaystyle\lim_{x\to0}(1+2x)^{1/x}$.

3. (20 points) Evaluate each of the following integrals.

(a) $\displaystyle\int_0^{\sqrt{3}/2} \frac{dx}{1+4x^2}$

(b) $\displaystyle\int_3^6 \frac{5x-1}{x^2-x-2}\,dx.$

4. (20 points) Calculate the following integrals.

(a) $\int \frac{\ln x}{x^3} \, dx.$

(b) $\int \frac{x^2}{\sqrt{1 - x^2}} \, dx.$

5. (10 points) A sum of money is invested at a fixed rate of interest, compounded continuously. The investment doubles in ten years. Find the interest rate.

6. (10 points) If g is a continuous function satisfying $\int_1^{e^2} g(x)\,dx = 2002$, find $\int_0^1 e^{2x}\, g(e^{2x})\,dx$.