| Math  | 126: | Calc  | ulus | s II |
|-------|------|-------|------|------|
| Final | Dece | ember | 17,  | 2002 |

| Name:      |  |
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| Instructor |  |

There are 12 problems on 10 pages (including the cover page) worth a total of 140 points. You start with 10 points. Unless otherwise specified, each part of a problem is worth the same number of points.

You may use a calculator if you wish.

To receive full credit for a problem, you must *give an exact answer* unless otherwise specified. To receive partial credit on a problem, you must *show your work and all important steps*. No credit will be given for an answer if no work is shown.

- 1.
- 2. \_\_\_\_\_
- 3. \_\_\_\_\_
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- 10.
- 11. \_\_\_\_\_
- 12. \_\_\_\_\_
- + 10

Total \_\_\_\_\_

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1. (15 points) Evaluate the integral:

$$\int_{1}^{4} e^{\sqrt{x}} \, dx.$$

2. (10 points) Find  $\lim_{x \to \infty} \left( \frac{x+1}{x} \right)^{4x}$ .

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3. (10 points) Find the centroid of the region enclosed by the graphs of y=4 and  $y=x^2$ .

4. (15 points) Solve the initial value problem:

$$(1+x^2)y' + xy^2 = x, y(0) = 0.$$

Hint: This is a separable equation.

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5. (10 points) A bacteria culture starts with 600 bacteria and grows at a rate proportional to its size. If after 3 hours there are 12,000 bacteria, how long will it take until the population reaches 45,000.

6. (10 points) For which p > 0 is the series

$$\sum_{n=1}^{\infty} \frac{2 + e^{-n}}{n^p}$$

convergent? For which is it divergent? Justify your answer.

7. (10 points) Use the integral test to examine the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$$

for convergence or divergence.

8. (10 points) Find the radius of convergence and the interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{x^n}{n^5}$ .

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- 9. (15 points)
  - (a) Find the Maclaurin series of  $f(x) = x^2 \cos x$ .

(b) Find  $f^{(2002)}(0)$ .

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- 10. (15 points)
  - (a) Find a power series for  $\int_0^x e^{-t^2} dt$ .

(b) Evaluate  $\int_0^1 e^{-t^2} dt$  with an error less than  $10^{-2}$ .

- 11. (10 points) Consider the parametric curve  $x=2\cos t,\ y=3\sin t,\ 0\leq t\leq\pi.$ 
  - (a) Sketch this curve.

(b) Set up an integral to compute the length of this curve. Do  ${\bf NOT}$  evaluate the integral.

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12. (10 points) Find the area of the region that lies inside the curve  $r=2-2\cos\theta$  but outside the curve r=2.