Math 126: Calculus II
Final December 17, 2002

Name: $\qquad$
Instructor: $\qquad$

There are 12 problems on 10 pages (including the cover page) worth a total of 140 points. You start with 10 points. Unless otherwise specified, each part of a problem is worth the same number of points.

You may use a calculator if you wish.
To receive full credit for a problem, you must give an exact answer unless otherwise specified. To receive partial credit on a problem, you must show your work and all important steps. No credit will be given for an answer if no work is shown.

1. $\qquad$
2. 
3. $\qquad$
4. $\qquad$
5. $\qquad$
6. $\qquad$
7. $\qquad$
8. $\qquad$
9. $\qquad$
10. $\qquad$
11. $\qquad$
12. $\qquad$
$+\quad 10$

Total $\qquad$

Name:

1. (15 points) Evaluate the integral:

$$
\int_{1}^{4} e^{\sqrt{x}} d x
$$

2. (10 points) Find $\lim _{x \rightarrow \infty}\left(\frac{x+1}{x}\right)^{4 x}$.

Name:
3. (10 points) Find the centroid of the region enclosed by the graphs of $y=4$ and $y=x^{2}$.

Name:
4. (15 points) Solve the initial value problem:

$$
\left(1+x^{2}\right) y^{\prime}+x y^{2}=x, \quad y(0)=0
$$

Hint: This is a separable equation.

Name: $\qquad$
5. (10 points) A bacteria culture starts with 600 bacteria and grows at a rate proportional to its size. If after 3 hours there are 12,000 bacteria, how long will it take until the population reaches 45,000.
6. (10 points) For which $p>0$ is the series

$$
\sum_{n=1}^{\infty} \frac{2+e^{-n}}{n^{p}}
$$

convergent? For which is it divergent? Justify your answer.

Name: $\qquad$
7. (10 points) Use the integral test to examine the series

$$
\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3}}
$$

for convergence or divergence.
8. (10 points) Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^{n}}{n^{5}}$.

Name:
9. (15 points)
(a) Find the Maclaurin series of $f(x)=x^{2} \cos x$.
(b) Find $f^{(2002)}(0)$.

Name:
10. (15 points)
(a) Find a power series for $\int_{0}^{x} e^{-t^{2}} d t$.
(b) Evaluate $\int_{0}^{1} e^{-t^{2}} d t$ with an error less than $10^{-2}$.

Name: $\qquad$
11. (10 points) Consider the parametric curve $x=2 \cos t, y=3 \sin t, 0 \leq t \leq \pi$.
(a) Sketch this curve.
(b) Set up an integral to compute the length of this curve. Do NOT evaluate the integral.

Name: $\qquad$
12. (10 points) Find the area of the region that lies inside the curve $r=2-2 \cos \theta$ but outside the curve $r=2$.

