amsppt

Name: ______________________ **Professor:** A.Alexandrou Himonas■

Mathematics 165
Honors Calculus
Fall Semester
Final Exam
December 16, 1991, 1:45 - 3:45 PM, Comp 328

This Examination contains eight problems worth a total of 150 points each problem worth 20 points,except the last which worths 10 points, on 10 sheets of paper including the front cover and one extra sheet on the back. Do all your work in this booklet and show your computations. Calculators, books and notes are not allowed.

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Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

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GOOD LUCK and HAPPY HOLIDAYS

1. i) Compare the derivative of the function

$$f(x) = 1 - x^5 + x1 + x^2 + \sin(x^2 + 3) + \int_{x^2}^{1+x^4} \sqrt{2 + \cos t}$$

ii) Compute the integral

$$I = \int_0^3 [x]x dx$$

2. i) Let $f(x) = 5x \sin 1x$. Use the definition of the limit to show that $\lim_{x \to 0} f(x) = 0$.

ii) Let $f(x) = 1x, x > 0$. Use the definition of the derivative to show that

$$f(x) = -1x^2.$$

3. Suppose that the function $y = f(x)$ is defined for $-\infty < x < \infty$ and has first, second, and third derivatives for each x. Use the letters corresponding to the choices given below to form true statements. i) If $f'(x) < 0$ for each x then f is ________________.

ii) If $f''(x) > 0$ for each x then f is ________________.

iii) If $f'(1) = 0$ and $f''(1) < 0$ then f has a ________________ at $x = 3$.

iv) If $f''(x) < 0$ for $x < 3$ and $f''(x) > 0$ for $x > 3$ then f has a ________________ at $x = 3$.

v) If $f''(x) = 0$ for all x then the graph of f is ________________.

CHOICES

(a) concave upwards

(b) concave downwards

(c) relative or local maximum

(d) relative or local minimum

(e) point of inflection

(f) straight line

(g) parabola

(h) increasing

(i) decreasing

4. The length l of a rectangle is decreasing at the rate of 2 cm/s and the width w is increasing at the rate of 2 cm/s. When $l = 12cm$ and $w = 5cm$, find the rates of change of

(a) the area ________________

(b) the perimeter ________________

(c) the lengths of the diagonals of the rectangle ________________

Which of these quantities are increasing and which are decreasing?

increasing quantities ________________

decreasing quantities ________________

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5) Find the dimensions r and h of the cone of maximum volume described symmetrically in a hemisphere of radius 1 as illustrated. Note: the volume of a cone equals one third of its base area times its height.

6) Compute the area of the shaded region A illustrated below. a) The area is ________________

b) The volume is ________________

7) Intermediate Value Theorem. i) If f is continuous on $[a, b]$ and y_0 is a number between $f(a)$ and $f(b)$, $f(a) \neq f(b)$, then there exists an $x_0 \in (a, b)$ such that $f(x_0) =$

Complete the statement of each of the following theorems:

ii) **The Mean Value Theorem** If $y = f(x)$ is continuous at each point of $[a, b]$ and differentiable at each point of $[a, b]$, then there is at least one number c between a and b for which $f(b) - f(a)b - a =$

iii) **The Second Fundamental Theorem of Integral Calculus.** If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)dx =$$

iv) **The First Fundamental Theorem of Integral Calculus.** If f is continuous on $[a, b]$ then

$$F(x) = \int_a^x f(t)dt$$

is differentiable at every point x in $[a, b]$ and

$$dFdx = ddx \int_a^x f(t)dt =$$

v) **Leibniz's Rule** If f is continuous on $[a, b]$, and $u(x)$ and $v(x)$ are differentiable functions of x whose values lie in $[a, b]$, then

$$ddx \int_{u(x)}^{v(x)} f(t)dt =$$

8) Do either (i) or (ii)

(i) Prove any one of the theores we did in class this semester:

(ii) Let $f[0, 2] \to R$ be a function which is continuous at 1. If $f(1) > 0$ then show that there is $\delta > 0$ such that $f(x) > 12f(1)$, for any $x \in (12 - \delta, 12 + \delta)$.

Answer: ______________________________