Solutions to Math 165 Final Exam

1 i.

$$f'(x) = -5x^4 + 1 + x^2 - x \cdot 2x(1+x^2)^2 + 2x\cos(x^2+3) + \sqrt{2+\cos(1+x^4)(1+x^4)} - \sqrt{2+\cos x^2}\ 2x$$

1 ii. I

$$I = \int_1^2 x dx + \int_2^3 2x dx = x^2 2 \,\Big|_1^2 + x^2\Big|_2^3 = (2 - 12 + (9-4) = 32 + 5 = 132.$$

2 i. Let $\xi > 0$. We want to find $\delta > 0$ such that $0 < |x| < \delta \Rightarrow |f(x)| < \xi$. To have $|f(x)| = |\xi x \sin 1x| < 5 \cdot \delta \cdot 1 < \xi$ we must choose $\delta < \xi 5$, say $\delta = \xi 6$.

2 ii. We have

$$f(x+h) - f(x)h = 1x + h - 1xh = x - (x-h)x(x+h)h = -1x(x+h) \to -1x^2 as\ h \to 0.$$

Thus $f'(x) = -1x^2$.

3. i) f is decreasing. ii) f is concave upwards. iii) f has a local maximum at $x = 1$. iv) f has a point of inflection at $x = 3$. v) f is a straight line.

4. a) $A = lw$. Thus $A' = l'n + lw'$ and $A' = -2.5 + 12.2 = 4$ b) Perimeter $P = 2l + 2w$. Thus, $P' = 2l' + 2w'$ and $P' = 2(-2) + 2 \cdot 2 = 0$. c) The length of diagonal $d = \sqrt{l^2 + w^2}$ or $d^2 = l^2 + w^2$. Thus $dd' = ll' + ww'$ and $13d' = 12 \cdot (-2) + 5 \cdot 2 = -14$ **or** $d' = -1413$. $Increasing quantities = area \quad Decreasing quantities = diagonals.$

5. Volume $V = 13\pi r^2 h$. Since $r^2 = 1 - h^2$, we have $V = V(h) = 13\pi h(1-h^2), 0 \leq h \leq 1$. We have $V'(h) = 13\pi\left[1 - 3h^2\right]$. Thus $V' = 0$ iff $3h^2 = 1$ or $h^2 = 13$ or $h = \pm 1\sqrt{3}$. Since $h \geq 0$ we have $h = 1\sqrt{3}$. Since $V''(h) = 13\pi(-6h) < 0$ at $h = 1\sqrt{3}$ we have that the volume is maximum when $h = 1\sqrt{3}$.

6.
i) $A = \int_{-1}^1 \left[2 - x^2 - 1\right] dx = 2\int_0^1 (1-x^2)dx = 2(x - x^3 3)|_0^1 = 2(1 - 13) = 43$
ii)

$$V = \pi \int_{-1}^1 [2^2 - (x^2+1)^2]dx = 2\pi \int_{-1}^1 (3 - x^2 - 2x)dx = 2\pi(3x - x^3 3 - x^2)|_0^1 = 2\pi(3 - 13 - 1) = 103\pi$$

i) The area is 43. ii) The volume is $10\pi 3$.

7.
i) $f(x_0) = y_0$

ii) $f(b) - f(a)b - a = f'(c)$

iii)$dFdx = ddx \int_a^x f(t)dt = f(x)$

iv) $\int_a^b f(x)dx = F(b) - F(a)$
v) $ddx \int_{g(x)}^{h(x)} f(t)dt = f(h(x))h'(x) - f(g(x))g'(x)$

8. ii) Since f is continuous at $x = 1$ for $\xi = 12f(1)$ there exist $\delta > 0$ such that if $|f(x) - f(1)| < \xi = 12f(1)$

or $-12f(1) < f(x) - f(1) < 12f(1)$

or

$$f(1) - 12f(1) < f(x) < 12f(1) + f(1)$$

or

$$12f(1) < f(x) < 32f(1).$$

Thus, $f(x) > 12f(1)$ for $|x - 1| < \delta$.