Math 165: Honors Calculus I Name:____________________________
Exam II *Dec. 1, 1994*

There are 9 problems worth from 5 to 20 points for a total of 120 points.

1. (20 pts) Define the following concepts precisely.

   a) $\lim_{x \to p} f(x) = A$.

   b) A function f is continuous at p.

   c) The inverse of a 1-to-1 function $f(x)$.

   d) The span of a bounded function f on $[a, b]$.

2. (20 pts) State the following theorems precisely.

a) Intermediate Value Theorem.

b) Mean Value Theorem for Integrals.

c) Extreme Value Theorem.

d) Squeezing Principle.

e) Chain Rule.

3. (20 pts) Compute the following limits or prove they do not exist.

a) $\lim_{x \to a} \dfrac{x^2 - a^2}{x^3 - a^3}$.

b) $\lim_{x \to 0} x \sin \dfrac{1}{x}$.

c) $\lim_{x \to 1} \dfrac{\tan(x-1)}{\tan(2x-2)}$.

d) $\lim_{x \to 3^-} x - [x]$.

e) $\lim_{x \to 0} x\sqrt{1 + \dfrac{1}{x^2}}$.

4. (20 pts) Compute the derivatives of the following functions. (Do not simplify.)

a) $f(x) = x^{2/3} + \frac{1}{x}$.

b) $f(x) = (5x^2 - 3)(x^3 + 1)^4$

c) $f(x) = \dfrac{x-1}{(x-2)(x-3)}$

d) $f(x) = \sin^3(x^2 - 1)$

e) $f(x) = u \circ v \circ w(x)$

5. (10 pts) Let f be an integrable function on $[a, b]$ and let $F(x) = \int_a^x f(t)\, dt$. Prove that F is continuous at $c \in [a, b]$.

6. (5 pts) Prove that the polynomial $f(x) = x^5 - 5x + 1$ has a root in $[0, 1]$.

7. (5 pts) Determine the equation of the line tangent to the graph of the function $f(x) = \dfrac{x}{\sqrt{1 + x^2}}$ at the point $(1, 1/\sqrt{2})$.

8. (10 pts)

a) Prove that if f is differentiable at p then f is continuous at p.

b) Give an example of a function that is continuous p but not differentiable at p. (Give a proof of this).

9. (10 pts) Let $f(x) = \sqrt{x}$ and let $p > 0$.

a) Using the definition of continuity, prove that $f(x)$ is continuous at p.

b) Using the definition of the derivative, prove that $f'(p) = \dfrac{1}{2\sqrt{p}}$.