Math 165: Honors Calculus I Name:____________________________
Quiz 4 *Sept. 29, 1994*

1. a) State the least upper bound axiom (Axiom #10).

   b) Prove that if A is a non-empty subset of B and if B has an upper bound, then $\sup A \leq \sup B$.

   c) Let $A = \{1/n \,|\, n \in\}$. Compute (or prove it doesn't exist): $\sup A$ and $\inf A$.

2. Let f be a bounded function on $[a, b]$.

   a) Define the lower integral, $\underline{I}(f)$, and the upper integral, $\overline{I}(f)$.

b) State a condition on $\underline{I}(f)$ and $\overline{I}(f)$ that is equivalent to f being integrable.

c) Define what it means for f to be monotone on $[a, b]$.

3. Let f be increasing on $[a, b]$.

a) Describe how to approximate $\int_a^b f(x)\,dx$ by dividing $[a, b]$ into n subintervals of equal length.

b) If $f(x) = x + 1/x$ on $[1, 2]$, how large must n be in order that the approximation in part a) is within 0.001 of the actual value of $\int_1^2 f(x)\,dx$.