

Math 165: Honors Calculus I
Quiz 9 Nov. 17, 1994

Name: _____

1. a) State the BOUNDEDNESS THEOREM FOR CONTINUOUS FUNCTIONS.

b) State the EXTREME-VALUE THEOREM FOR CONTINUOUS FUNCTIONS.

c) Use a) to prove b). Hint: Let $M = \sup f$, $g(x) = M - f(x)$, and consider $1/g(x)$.

2. Let $f(x)$ be continuous on $[a, b]$ and suppose $0 < f(x) < 1$ for all $x \in [a, b]$. Prove that there is a positive integer n such that

$$\frac{1}{n} \leq f(x) \leq 1 - \frac{1}{n}$$

for all $x \in [a, b]$

3. Let $f(x)$ be continuous on $[a, b]$ and let $p \in [a, b]$. Prove that for any $\varepsilon > 0$ there exists a subinterval of $[c, d] \subset [a, b]$ with $p \in (c, d)$ such that the span of f on $[c, d]$ is $< \varepsilon$.