1. a) (5 pts) Define the positive integers, .

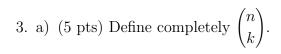
b) (5 pts) Define the rational numbers, .

c) (5 pts) Using only the axioms for the real numbers, prove that $a \cdot 0 = 0$ for all $a \in$.

2. a) (5 pts) State the Principle of Mathematical Induction.

b) (5 pts) Prove by induction that for $n \in$

$$(x-1)\sum_{k=0}^{n-1} x^k = x^n - 1$$



b) (5 pts) State the BINOMIAL THEOREM.

c) (5 pts) Find the coefficient of x^{15} in $(x^3 + 2)^{20}$ (give the prime factorization of this coefficient).

4. a) (2 pts) Define what it means for a set S of real numbers to be bounded from below.

b) (3 pts) Define inf S.

c) (5 pts) Let $S = \left\{ \frac{n}{n^2 + 1} \mid n \in \right\}$. Prove that $\inf S = 0$.

- 5. Give precise definitions of the following.
 - a) (5 pts) A step function s on [a, b].

b) (5 pts) $\int_a^b s(x) dx$ where s is a step function on [a, b].

c) (5 pts) The upper integral of f on $[a,b], \overline{I}(f)$.

d) (5 pts) An integrable function f on [a, b].

6. (5 pts) Find an approximation for $\int_0^1 \frac{1}{1+x^3} dx$ that is accurate to within ± 0.075 .

7.	Give pre	cise sta	tements	of the	following	theorems	for	integrals.
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a) (5 pts) The Linearity Theorem.

b) (5 pts) The Expansion/Contraction Theorem.

8. Evaluate the following integrals (justify your answers!).

a) (5 pts)
$$\int_{-1}^{2} |3x^2 - 6x| dx$$
.

b) (5 pts)
$$\int_1^3 (x-2)^{17} dx$$
.

c) (5 pts)
$$\int_{-3}^{3} \frac{x^5}{\sqrt{2+x^4}} dx$$
.

9. a) (5 pts) Find an expression for the area between the graphs of $f(x) = x^2$ and g(x) = 3x-2 on the interval [0, 4]. Write the answer as a sum of integrals without absolute values—do *not* evaluate the integrals.

b) (5 pts) Find the average value of the function $f(x) = \sqrt{x}$ on the interval [0, a].