

Math 165: Honors Calculus I

Name: _____

Exam I *Sept. 28, 1995*

There are 9 problems worth of total of 110 points.

1. a) (5 pts) Define the positive integers, .

b) (5 pts) Define the rational numbers, .

c) (5 pts) Using only the axioms for the real numbers, prove that $a \cdot 0 = 0$ for all $a \in \mathbb{R}$.

2. a) (5 pts) State the PRINCIPLE OF MATHEMATICAL INDUCTION.

b) (5 pts) Prove by induction that for $n \in$

$$(x - 1) \sum_{k=0}^{n-1} x^k = x^n - 1$$

3. a) (5 pts) Define completely $\binom{n}{k}$.

b) (5 pts) State the BINOMIAL THEOREM.

c) (5 pts) Find the coefficient of x^{15} in $(x^3 + 2)^{20}$ (give the prime factorization of this coefficient).

4. a) (2 pts) Define what it means for a set S of real numbers to be bounded from below.

b) (3 pts) Define $\inf S$.

c) (5 pts) Let $S = \left\{ \frac{n}{n^2 + 1} \mid n \in \mathbb{N} \right\}$. Prove that $\inf S = 0$.

5. Give precise definitions of the following.

a) (5 pts) A step function s on $[a, b]$.

b) (5 pts) $\int_a^b s(x) dx$ where s is a step function on $[a, b]$.

c) (5 pts) The upper integral of f on $[a, b]$, $\bar{I}(f)$.

d) (5 pts) An integrable function f on $[a, b]$.

6. (5 pts) Find an approximation for $\int_0^1 \frac{1}{1+x^3} dx$ that is accurate to within ± 0.075 .

7. Give precise statements of the following theorems for integrals.

a) (5 pts) THE LINEARITY THEOREM.

b) (5 pts) THE EXPANSION/CONTRACTION THEOREM.

8. Evaluate the following integrals (justify your answers!).

a) (5 pts) $\int_{-1}^2 |3x^2 - 6x| dx$.

b) (5 pts) $\int_1^3 (x - 2)^{17} dx$.

c) (5 pts) $\int_{-3}^3 \frac{x^5}{\sqrt{2 + x^4}} dx$.

9. a) (5 pts) Find an expression for the area between the graphs of $f(x) = x^2$ and $g(x) = 3x - 2$ on the interval $[0, 4]$. Write the answer as a sum of integrals without absolute values—do *not* evaluate the integrals.

b) (5 pts) Find the average value of the function $f(x) = \sqrt{x}$ on the interval $[0, a]$.