**Math 165: Honors Calculus I** Name:____________________

**Exam II** *Nov. 16, 1995*

There are 8 problems worth a total of 110 points.

1. (20 pts) Define the following concepts precisely.

   a) $\lim\limits_{x \to p} f(x) = A$.

   b) A function $f(x)$ is continuous at $p$.

   c) The inverse of a 1-to-1 function $f(x)$.

   d) The derivative $f'(x)$ of a function $f(x)$.

2. (20 pts) State the following theorems precisely.

a) Intermediate Value Theorem.

b) Mean Value Theorem for Integrals.

c) Extreme Value Theorem.

d) Squeezing Principle.

3. (20 pts) Evaluate the following limits (justify your answers!) or prove they do not exist.

a) $\displaystyle\lim_{x\to 1} \frac{\sqrt{x^2-2x+5}-2}{(x-1)^2}$.

b) $\displaystyle\lim_{x\to 2}(x-2)\cos(\frac{1}{x-2})$.

c) $\displaystyle\lim_{x\to 0} \frac{\tan(3x)}{\tan(2x)}$.

d) $\displaystyle\lim_{x\to 4^-} x-[x]$.

e) $\displaystyle\lim_{x\to 0} x\left|\frac{1}{x}-1\right|$.

4. (10 pts) Compute the derivatives of the following functions. (Do not simplify.)

a) $f(x) = x^{4/5} + \dfrac{1}{x^2}.$

b) $f(x) = \dfrac{x-3}{(x-1)(x-2)}$

c) $f(x) = x\cos(x)\sin(x)$

5. (10 pts) Let $f$ be an integrable function on $[a, b]$ and let $F(x) = \int_a^x f(t)\,dt$. Prove that $F$ is continuous at $c \in [a, b]$.

6. (10 pts)

a) State Bolzano's Theorem.

b) Prove that the polynomial $f(x) = x^5 - 5x + 1$ has a root in $[0, 1]$.

7. (10 pts)

a) State the SMALL SPAN THEOREM.

b) Use a) to prove that if a function $f$ is continuous on $[a, b]$ then $f$ is integrable on $[a, b]$.

8. (10 pts) Let $f(x) = \dfrac{1}{x}$ and let $p > 0$.

a) Using the definition of continuity, prove that $f(x)$ is continuous at $p$.

b) Using the definition of the derivative, prove that $f'(p) = -\dfrac{1}{p^2}$.