**Math 165: Honors Calculus I** Name:____________________________

**Quiz 4** *Sept. 21, 1995*

1. a) Define the supremum, $\sup A$, of a subset of real numbers $A \subset$.

   b) Prove that if $A$ is a non-empty subset of $B$ and if $B$ has an upper bound, then $\sup A \leq \sup B$.

   c) Let $A = \{1/n \,|\, n \in\}$. Compute (or prove it doesn't exist): $\sup A$ and $\inf A$.

2. Let $f$ be a bounded function on $[a, b]$.

   a) Define the lower integral of $f$, $\underline{I}(f)$, and the upper integral of $f$, $\overline{I}(f)$.

   b) State a condition on $\underline{I}(f)$ and $\overline{I}(f)$ that is equivalent to $f$ being integrable.

3. a) Define what it means for $f$ to be piecewise monotone on $[a, b]$.

   b) Let $f$ be increasing on $[a, b]$. Describe how to approximate $\int_a^b f(x)\,dx$ by dividing $[a, b]$ into $n$ subintervals of equal length.