Math 165: Honors Calculus I Name:________________________

Quiz 9 *Nov. 9, 1995*

1. a) State the BOUNDEDNESS THEOREM FOR CONTINUOUS FUNCTIONS.

   b) Use a) to prove that if f is continuous on $[a, b]$ then f has a maximum on $[a, b]$. (Hint: Let $M = \sup_{[a,b]} f$, $g(x) = M - f(x)$, and consider $1/g(x)$.)

2. Let $f(x)$ be continuous on $[a, b]$ and suppose $0 < f(x) < 1$ for all $x \in [a, b]$. Prove that there is a positive integer n such that

$$\frac{1}{n} \leq f(x) \leq 1 - \frac{1}{n}$$

for all $x \in [a, b]$

3. Let $f(x)$ be continuous on $[a, b]$ and let $p \in [a, b]$. Prove that for any $\varepsilon > 0$ there exists a neighbohood of p, $N(p) \subset [a, b]$, such that the span of f on $N(p)$ is $< \varepsilon$.