

**Math 165: Honors Calculus I**  
**Assignment 16** *Oct. 9, 1998*

1. Show that  $\lim_{x \rightarrow 0^+} \sqrt{x} = 0$ .  
(Hint:  $0 < x < \delta \Rightarrow 0 < \sqrt{x} < \sqrt{\delta}$ .)
2. Show that  $\lim_{x \rightarrow 0^+} x^{1/n} = 0$  for  $n \in \mathbb{N}$ .
3. Show that  $\lim_{x \rightarrow 0^-} x^{1/n} = 0$  for  $n$  an odd positive integer.
4. Let  $f(x) = \frac{|x|}{x}$  for  $x \neq 0$ . Use one-sided limits to show that  $\lim_{x \rightarrow 0} f(x)$  does not exist.
5. Let  $f(x) = x^2$  and let  $p \in \mathbb{R}$ .
  - a) Show that if  $|x - p| < 1$  then  $|x| < |p| + 1$ .  
(Hint: consider  $||x| - |p|| \leq |x - p|$ .)
  - b) Show that if  $|x - p| < 1$  then  $|x^2 - p^2| < (2|p| + 1)|x - p|$ .
  - c) Show that  $\lim_{x \rightarrow p} x^2 = p^2$ .  
(Hint: try  $\delta = \min(1, \frac{\epsilon}{2|p| + 1})$ .)