

**Math 165: Honors Calculus I**  
**Assignment 21** *Nov. 6, 1998*

1. Let  $f$  be continuous and positive on  $[a, b]$ . Prove that there exists a constant  $c > 0$  such that  $f(x) \geq c$  for all  $x \in [a, b]$ .
  
2. Give an example of a function  $g$  on  $[0, 1]$  such that  $g(x) > 0$  for all  $x \in [0, 1]$ , but there is no constant  $c > 0$  such that  $g(x) \geq c$  for all  $x \in [0, 1]$ .
  
3. Let  $f$  be continuous on  $[a, b]$  and let  $(p, q)$  be any point in the plane. Prove that there exists a point on the graph of  $f$  that is closest to  $(p, q)$ . (Hint: What is the distance from  $(p, q)$  to  $(x, f(x))$ ?)