Math 165: Honors Calculus I Name:____________________________
Exam II *Nov. 19, 1998*

There are 8 problems worth a total of 110 points.

1. (20 pts) Define the following concepts precisely.

   a) $\lim_{x \to p} f(x) = A$.

   b) A function f is continuous at p.

   c) The inverse of a 1-to-1 function $f(x)$.

   d) The derivative $f'(x)$ of a function $f(x)$.

2. (20 pts) State the following theorems precisely.

   a) Basic Limit Theorems.

   b) Intermediate Value Theorem.

   c) Extreme Value Theorem.

   d) Mean Value Theorem for Integrals.

3. (20 pts) Evaluate the following limits (justify your answers!) or prove they do not exist.

   a) $\lim_{x \to 1} (x-1) \sin \frac{x}{x-1}$

   b) $\lim_{x \to 0} \frac{\cos(x) - 1}{x^2}$

   c) $\lim_{x \to 2^-} x^3 - [x^3]$

   d) $\lim_{x \to 0} x \left| 1 + \frac{1}{x} \right|$

4. (15 pts)

a) Let $f(x) = x^{2/3} + \dfrac{1}{x}$, $x > 0$. Compute $f'(x)$.

b) Let a and b be constants, not both zero, and let $f(x) = \dfrac{ax+b}{a-bx}$. Show that $f'(x) > 0$ for $x \neq a/b$.

c) Let $f(x) = (ax+b)\cos(x) + (cx+d)\sin(x)$. Determine values of the constants a, b, c, d, such that $f'(x) = x\sin(x)$.

5. (10 pts) Let f be an integrable function on $[a,b]$ and let $F(x) = \int_a^x f(t)\,dt$. Prove that F is continuous at $c \in [a,b]$.

6. (5 pts) Let $n \geq 2$ be a positive integer. Prove that the polynomial $f(x) = x^n - nx + 1$ has a root in $[0,1]$.

7. (10 pts) Find the largest interval I containing $x = 1$ on which the function $f(x) = \dfrac{1}{1+x^2}$ has an inverse. Give a formula for the corresponding inverse function, $f^{-1}(x)$, as a function of x and determine its domain.

8. (10 pts)

a) Using the definition of continuity, prove that $f(x) = x^2$ is continuous at any real number p.

b) Using the definition of the derivative, prove that the derivative of $f(x) = \sqrt{x}$ is $f'(x) = \dfrac{1}{2\sqrt{x}}$.