

Math 165: Honors Calculus I
Quiz 4 Sept. 24, 1998

Name: _____

1. a) Define the supremum, $\sup A$, of a subset of real numbers $A \subset \mathbb{R}$.

b) Let $A = \left\{ 1 - \frac{1}{n} \mid n \in \mathbb{N} \right\}$. Prove $\sup A = 1$.

2. Let $f(x) = \frac{1}{x}$ for $x \in [1, 2]$. Find step functions s_n and t_n on $[1, 2]$:

- (a) that are constant on n open subintervals of *equal length*,
- (b) satisfy $s_n(x) \leq f(x) \leq t_n(x)$ for all $x \in [1, 2]$,
- (c) are the best approximation to $f(x)$ by step functions from below and above, respectively, subject to constraints (a) and (b).

(d) Calculate $\int_1^2 s_n(x) dx$ and $\int_1^2 t_n(x) dx$, and conclude that

$$\sum_{k=1}^n \frac{1}{n+k} < \int_1^2 \frac{1}{x} dx < \sum_{k=0}^{n-1} \frac{1}{n+k}$$

3. Calculate $\int_0^2 |(x-1)^5| dx$ using only the basic properties of integrals covered so far in the course.