

Math 165: Honors Calculus I
Quiz 6 Oct. 15, 1998

Name: _____

1. Give precise definitions for the following.

a) $\lim_{x \rightarrow p} f(x) = A$.

b) $f(x)$ is continuous at p .

2. Let $f(x) = [x]^2$ where $[x]$ is the greatest integer $\leq x$. Use one-sided limits to show that $\lim_{x \rightarrow 2} f(x)$ does not exist.

3. Suppose $f(x)$ is a function that has the following property:

$$\text{if } -0.25 < x < 0.25 \text{ then } 1 - |x|^3 < f(x) < 1 + |x|^3$$

The graph of $f(x)$ would thus lie between the curves $y = 1 - |x|^3$ and $y = 1 + |x|^3$, at least for x near 0, and might look something like the following:

Prove, *using the definition of limit*, that $\lim_{x \rightarrow 0} f(x) = 1$.