

Math 165: Honors Calculus I
Quiz 9 Nov. 12, 1998

Name: _____

1. a) State the BOUNDEDNESS THEOREM FOR CONTINUOUS FUNCTIONS.

b) Use a) to prove that if f is continuous on $[a, b]$ then f has a maximum on $[a, b]$. (Hint: Let $M = \sup_{[a,b]} f$, $g(x) = M - f(x)$, and consider $1/g(x)$.)

2. Let $f(x)$ be continuous on $[a, b]$ and let $p \in (a, b)$. Prove that for any $\varepsilon > 0$ there exists a neighborhood of p , $N(p) \subset [a, b]$, such that the span of f on $N(p)$ is $< \varepsilon$.

3. Suppose $f(x)$ and $g(x)$ are continuous functions on $[a, b]$ satisfying $f(x_1) < g(x_2)$ for any $x_1, x_2 \in [a, b]$. Prove that there is a *constant* c such that $f(x) < c < g(x)$ for all $x \in [a, b]$.