Math 165: Honors Calculus I Name:________________________
Exam II *November 11, 1999*

There are 7 problems worth a total of 110 points.

1. (25 pts) Define the following precisely.

   a) The area between the graphs of two integrable functions on $[a, b]$.

   b) The functions $\sin(x)$ and $\cos(x)$.

   c) The limit of a function $f(x)$ as x approaches p.

   d) A one-to-one function.

   e) The average value of an integrable function on $[a, b]$.

2. (25 pts) State the following theorems.

   a) Basic Limit Theorems

   b) The Squeezing Principle

   c) Bolzano's Theorem

   d) Intermediate Value Theorem

   e) Extreme Value Theorem

3. (20 pts) Evaluate the following limits (justify your answers!) or prove they do not exist.

a) $\lim\limits_{x \to 1} \dfrac{\sin(x^2 - 1)}{x - 1}$

b) $\lim\limits_{x \to 0} \dfrac{\sqrt{x^4 + 1} - 1}{x^4}$

c) $\lim\limits_{x \to 2} \sqrt{x^3 - \dfrac{1}{x^3}}$

d) $\lim\limits_{x \to 0^+} x^2\left(1 + \left[\dfrac{1}{x}\right]\right)$. (As usual, $[a]$ denotes the greatest integer $\leq a$.)

4. (10 pts) Assume $\lim_{x \to p} f(x) = A$ and $\lim_{x \to p} g(x) = B$. Using the definition of limit, prove that $\lim_{x \to p} (f(x) - g(x)) = A - B$.

5. (10 pts) Let f be continuous at p and let g be continuous at $q = f(p)$. Prove that $g \circ f$ is continuous at p.

6. (10 pts) Let f be integrable and positive on $[a, b]$. Show that the function $F(x) = \int_a^x f(t)\, dt$ is increasing on $[a, b]$.

7. (10 pts) Suppose the inverse of a function is $f^{-1}(x) = \sqrt{\dfrac{1}{\sqrt{x}} - 1}$ for $0 < x \le 1$. Determine the original function, $f(x)$, including its domain.