Math 165: Honors Calculus I Name:______________________________
Quiz 1 *September 2, 1999*

Axioms for the Real Numbers

A set $\mathbb{R}$, called the set of real numbers, is assumed to exist satisfying the ten axioms below.

The Field Axioms. Two operations on $\mathbb{R}$, *addition* and *multiplication*, are assumed to be defined, so that for each pair, x, $y \in \mathbb{R}$, there is a uniquely determined *sum*, $x + y \in \mathbb{R}$, and a uniquely determined *product*, $x \cdot y \in \mathbb{R}$, satisfying the following axioms.

- Axiom 1. $x + y = y + x$, $xy = yx$
- Axiom 2. $x + (y + z) = (x + y) + z$, $x(yz) = (xy)z$
- Axiom 3. $x(y + z) = xy + xz$
- Axiom 4. There exists distinct numbers 0 and 1 such that for every $x \in \mathbb{R}$, $x + 0 = x$ and $1 \cdot x = x$.
- Axiom 5. For every $x \in \mathbb{R}$, there is a $y \in \mathbb{R}$ such that $x + y = 0$.
- Axiom 6. For every $x \in \mathbb{R}$, $x \neq 0$, there is a $y \in \mathbb{R}$ such that $xy = 1$.

The Order axioms A subset $\mathbb{R}^+ \subset \mathbb{R}$, called the positive numbers, is assumed to exist satisfying the following axioms.

- Axiom 7. If x and y are in $\mathbb{R}^+$, so are $x + y$ and xy.
- Axiom 8. For every real $x \neq 0$, either $x \in \mathbb{R}^+$ or $-x \in \mathbb{R}^+$, but not both.
- Axiom 9. $0 \notin \mathbb{R}^+$.

The Completeness Axiom

- Axiom 10. For every non-empty subset $S \subset \mathbb{R}$ that is bounded above there is a $B \in \mathbb{R}$ that is the supremum of S, $B = \sup S$.

(1) Using only the axioms for the real numbers, prove the following statement: Given any real numbers a, b, there is a *unique* real number x such that $a + x = b$.

(2) Define an inductive set, S.

(3) Use the Principle of Mathematical Induction to prove $2^n > n$ for any positive integer n.