MATH 165: HONORS CALCULUS I
QUIZ 6, SOLUTION TO PROBLEM 3

Let $f(t) = (-1)^{[t]}$ where $[t]$ is the greatest integer less than or equal to t. Calculate $F(x) = \int_0^x f(t)\,dt$ for $0 \leq x \leq 4$ and plot its graph.

Solution.
The graph of $f(t)$ for $0 \leq t \leq 4$ is:

If $0 \leq x < 1$, then in the integral $F(x) = \int_0^x f(t)\,dt$ the variable t lies in the interval $0 \leq t \leq x < 1$, so $[t] = 0$. Thus, $f(t) = 1$ and $F(x) = \int_0^x 1\,dt = x$.
If $1 \leq x < 2$, then $1 \leq t \leq x$ implies $[t] = 1$ and $f(t) = -1$, so

$$F(x) = \int_0^1 f(t)\,dt + \int_1^x f(t)\,dt = \int_0^1 1\,dt + \int_1^x (-1)\,dt = 1 - x + 1 = 2 - x$$

If $2 \leq x < 3$, then $2 \leq t \leq x$ implies $[t] = 2$ and $f(t) = 1$, so

$$F(x) = \int_0^1 1\,dt + \int_1^2 (-1)\,dt + \int_2^x 1\,dt = 1 - 1 + x - 2 = x - 2$$

If $3 \leq x < 4$, then $3 \leq t \leq x$ implies $[t] = 3$ and $f(t) = -1$, so

$$F(x) = \int_0^1 1\,dt + \int_1^2 (-1)\,dt + \int_2^3 1\,dt + \int_3^x (-1)\,dt = 1 - 1 + 1 - x + 3 = 4 - x$$

The graph of $F(x)$ is: