Math 165: Honors Calculus I
Assignment 9 *Sept. 15, 1999*

1. Problem 13 on p. 45.

2. Let A and B be non-empty sets of real numbers bounded above and below, respectively.

   a) Show that for any $k < \sup A$ there exists a number $a \in A$ such that $k < a$.

   b) Show that for any $h > \inf B$ there exists a number $b \in B$ such that $b < h$.

3. Let A and B be sets of real numbers with $A \neq \emptyset$ and $A \subset B$. Prove:

   a) If A and B are bounded above, then $\sup A \leq \sup B$.

   b) If A and B are bounded below, then $\inf B \leq \inf A$.

4. For $0 \leq x \leq 1$ define

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

   Prove:

   a) If s and t are step functions satisfying $s(x) \leq f(x) \leq t(x)$ for all $0 \leq x \leq 1$, then $s(x) \leq 0$ and $t(x) \geq 1$, except possibly at partition points.

   b) $\underline{I}(f) = 0$, $\overline{I}(f) = 1$, and f is not integrable.