

Math 165: Honors Calculus I
Assignment 16 *Oct. 10, 1999*

1. Show that $\lim_{x \rightarrow 0^+} \sqrt{x} = 0$.
(Hint: $0 < x < \delta \Rightarrow 0 < \sqrt{x} < \sqrt{\delta}$.)
2. Show that $\lim_{x \rightarrow 0^+} x^{1/n} = 0$ for $n \in \mathbb{N}$.
3. Show that $\lim_{x \rightarrow 0^-} x^{1/n} = 0$ for n an odd positive integer.
4. Let $f(x) = \frac{|x|}{x}$ for $x \neq 0$. Use one-sided limits to show that $\lim_{x \rightarrow 0} f(x)$ does not exist.
5. Let $f(x) = x^2$ and let $p \in \mathbb{R}$.
 - a) Show that if $|x - p| < 1$ then $|x| < |p| + 1$.
(Hint: consider $||x| - |p|| \leq |x - p|$.)
 - b) Show that if $|x - p| < 1$ then $|x^2 - p^2| < (2|p| + 1)|x - p|$.
 - c) Show that $\lim_{x \rightarrow p} x^2 = p^2$.
(Hint: try $\delta = \min(1, \frac{\epsilon}{2|p| + 1})$.)