Math 165: Honors Calculus I
Assignment 20 *Nov. 3, 1999*

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6. Suppose f and g are functions such that $g(f(x)) = x$ for all x in the domain of f. Prove that f is one-to-one.

7. Let $f(x) = \int_1^x \frac{1}{t}\,dt$ for $x > 0$.

a) Prove that f is strictly increasing and $f^{-1}(0) = 1$.

b) Prove that $f(ab) = f(a) + f(b)$. (Hint: $\int_1^{ab} \frac{1}{t}\,dt = \int_1^{a} \frac{1}{t}\,dt + \int_a^{ab} \frac{1}{t}\,dt$, apply Theorem 1.19.)

c) Prove that $f^{-1}(a+b) = f^{-1}(a)f^{-1}(b)$.

8. Let f be strictly decreasing on $[a, b]$. Prove that the inverse f^{-1} is strictly decreasing on $[f(b), f(a)]$.

9. Let $f(x) = x^4 - 8x^2 + 8$. Determine the intervals on which $f(x)$ is strictly increasing and strictly decreasing and find the inverse of $f(x)$ on each of those intervals. Plot $f(x)$ and each of the inverses as functions of x. (Hint: simplify $f(x)$ by 'completing the square.')

10. Determine the largest interval containing 0 on which the functions $\sin(x)$ and $\tan(x)$ have inverses and plot the inverses on those intervals. Explain why there is no interval containing 0 (i.e., no interval of the form $[a, b]$ where $a < 0$ and $b > 0$) on which $\cos(x)$ has an inverse. Then find an interval on which $\cos(x)$ *does* have an inverse and plot the inverse there.