Math 165: Honors Calculus I
Assignment 21   *Nov. 8, 1999*

1. Let f be continuous and positive on $[a, b]$. Prove that there exists a constant $c > 0$ such that $f(x) \geq c$ for all $x \in [a, b]$.

2. Give an example of a function g on $[0, 1]$ such that $g(x) > 0$ for all $x \in [0, 1]$, but there is no constant $c > 0$ such that $g(x) \geq c$ for all $x \in [0, 1]$.

3. Let f be continuous on $[a, b]$ and let (p, q) be any point in the plane. Prove that there exists a point on the graph of f that is closest to (p, q). (Hint: What is the distance from (p, q) to $(x, f(x))$?)