UNIVERSITY OF NOTRE DAME

Mathematics 165: Honors Calculus I Fall 2000

Text: Calculus, Volume I, Second Edition, by Thomas Apostol, published by John Wiley & Sons, 1967. Most of the material in the Introduction and in Chapters 1-4 were covered.

Sets, definition of an ordered field, the natural numbers, integers, rational and irrational numbers. The Principle of Finite Induction. Summation notation. Absolute values. The Binomial Theorem.

Functions and their sums, products, and quotients. Properties of polynomials. Partitions and step functions. Definition and properties of integrals of step functions. Definition of the integral of a bounded function.

Upper, lower, least upper, and greatest lower bounds. The Least Upper Bound Axiom. The Archimedean Property of the real numbers. The sup and inf of sets formed by addition. Upper and lower integrals. The Epsilon Criterion for integrability.   Existence of the integral of a piecewise monotone function. Direct calculation of the integral of a power function. Properties of the integral.

Review of trigonometric functions. The two-sided limit of a function. Limits from one side. Continuity of a function at a point. Sum, product, quotient and squeeze limit theorems. Continuity of the integral as a function of its upper limit. Composition of functions and limits of composite functions.

Bolzano's Theorem. The Intermediate Value Theorem. One-to-one functions and their inverses. Continuity of the inverse of a continuous function. The Boundedness and Extreme Value Theorems. Existence of the integral of a continuous function. The Mean Value Theorem and Weighted Mean Value Theorem for integrals.

Definition of the derivative. Calulation of derivatives from the limit definition. The sum, product and quotient rules. The tangent line and the Tangent Line Approximation Theorem. Higher derivatives. The Chain Rule. Rates of change, related rates, and implicit differentiation.

Rolle's Theorem and the Mean Value Theorem.   Increasing and decreasing functions. The First and Second Derivative Tests for extrema. The Cauchy Mean Value Theorem. Convexity and its relation to the second derivative. Inflection points. Techniques of graphing.

Cardinality of sets. Card($\mathbf{N}$)=Card($\mathbf{Q}$)<Card($\mathbf{R}$)=Card($\mathbf{R}$x$\mathbf{R}$)<Card($\mathbf{P}(\mathbf{R})$)) (Just for fun, not for testing.)