MATH 165 TEST 1 (10/17/02)

NAME:

1) Write TRUE or FALSE for each of the following statements, in each case justifying your answer:

a) The number $2\sqrt{2}$ is irrational.

b) Let A, B be any two sets which are dense, in the sense that any (non-empty) open interval in $\mathbb{R}$ contains points from both A and B. Then the intersection of A and B is non-empty.

c) The set

$$S = \{x \in \mathbb{R} |\ |2x - 1| \leq 2,\ \ |x + 1| < 3\}$$

is empty.

d) If a function $f : [a, b] \to (0, \infty)$ is increasing, then the function $g = f^2$ is integrable.

e) Every bounded function $f : [a, b] \to \mathbb{R}$ is integrable.

2) Find the area of the region between the graphs of $f(x) = x^7$ and $g(x) = x^5$.

3) State and prove a theorem whose proof explicitly relies on the completeness axiom.

4) Prove the binomial formula

$$(a + b)^n = \Sigma_{k=0}^{n} \binom{n}{k} a^k b^{n-k}.$$