MATH 165 TEST 2 - PART A - (12/02/02)

NAME:

1) Write a proof of the following theorem (state the theorem here):

2) Write a short essay on the axiomatic approach to the concept of volume, including its relation to Cavalieri's principle.

MATH 165 TEST 2 - PART B - (12/04/02)

NAME:

3) Write TRUE or FALSE for each of the following statements, in each case justifying your answer:

a) If $f : [a, b] \to \mathbb{R}$ is such that f^2 is continuous, then f itself must be continuous.

b) If $f : [a, b] \to \mathbb{R}$ is bounded and integrable, then the function $F : [a, b] \to \mathbb{R}$ given by $F(x) = \int_a^x f(t)dt$ is continuous.

c) An application of Bolzano's theorem allows us to conclude that the equation $x^2 - x^4 = -1$ has a solution in the interval $[0, \frac{3}{2}]$.

d) If f is a continuous function on an *open* interval J, then the equation $f(x) = 0$ has only finitely many solutions in J.

e) If f is a continuous function on a *closed* interval J, then the equation $f(x) = 0$ has only finitely many solutions in J.

f) The stipulation that $f : \mathbb{R} \to \mathbb{R}$ is continuous at the point $a \in \mathbb{R}$ means, in symbols:

$$\forall \epsilon > 0 \ \ \exists \delta > 0 : \ \ |x - a| \leq \epsilon |f(x) - f(a)| \leq \delta.$$

g) Consider a function $f : [a, b] \to \mathbb{R}$ having the following property:

(†) For all $n \in \mathbb{Z}$ the equation $f(x) = n$ has at least one solution.

Then f must be discontinuous at some point of $[a, b]$.

h) One has

$$\lim_{x \to 0} |x|^{\frac{1}{10000}} \sin(\frac{1}{x}) = 0.$$

4) Using the double-angle formula, compute

$$\int_0^{\frac{\pi}{2}} \sin^2 x dx.$$

5) Compute

$$\lim_{t \to 0} \frac{\sin(\tan t)}{\sin t}.$$