amsppt

Honors Calculus 166
Spring 1992
Homework 2
Due January 31

A. From the book:

Page 325: $1_{i,iv,vi,viii,x}, 2_{i,iv}, 3, 4_{a,c,d,e}, 5_{i,iii,vi}7, 8, 9, 12, 16_{a,b,c}, 19, 23, 24, 29.$

B. Find the general solution to the following first order linear differential equations:

$$1.)\, y' - 2xy = x \qquad 2.)\, xy' - y = x,\ x > 0 \qquad 3.)\, y' + \cos xy' = \cos x.$$

C. Assuming that the population of the Earth now is 5 billion end that it grows at rate proportional to the current value with constant of proportionality equal to 0.02, find the population of the Earth at the year 2017. Take $e^{0.5}1.6487$.

D. A thirty years old person opens an Individual Retirement Account (IRA) with an initial investment of $\$3,000$ and then makes anual investments of $\$4,000$ thereafter continuously. Assuming an interest rate of 8% find the balance in the IRA at any time t. Also find the balance in the IRA when the person will be 60 years old. It is given that $e^{2.4}11.023$.

D. (From Boyce and Diprima Differential Equations book) **1.** Radium-226 has a half-life of 1620 years. Find the time period during which a body of this material is reduced to three-fourths of its original size. **2. Radiocarbon Dating.** An important tool in archeological research is radiocarbon dating. This is a means of determining the age of certain wood and plant remains, hence of animal or human bones or artifacts found buried at the same levels. The procedure was developed by the American chemist Willard Libby (1908-1980) in the early 1950's and resulted in his winning the Nobel prize for chemistry in 1960. Radiocarbon dating is based on the fact that some wood or plant remains contain residual amounts of carbon-14, a radioactive isotope of carbon. This isotope is accumulated during the lifetime of the plant and begins to decay at its death. Since the half-life of carbon-14 is long (approximately 5568 years[5]), measurable amounts of carbon-14 remain after many thousands of years. Libby showed that if even a tiny fraction of the original amount of carbon-14 is still present, then by appropriate laboratory measurements the proportion of theoriginal amount of carbon-14 that remains can be accurately determined. In other words, if $Q(t)$ is the amount of carbon-14 at time t and Q_0 is the original amount, then the ration $Q(t)Q_0$ can be determined, at least if this quantity is not too small. Present measurement techniques permit the use of this method for time periods up to about 100,000 years, after which the amount of carbon-14 remaining is only about $4X10^{-6}$ of the original amount.

(a) Assuming that Q satisfies the differential equation $Q' = -rQ$, determine the decay constant r for carbon-14.

(b) Find an expression for $Q(t)$ at any time t, if $Q(0) = Q_0$.

(c) Suppose that certain remains are discovered in which the current residual amount of carbon-14 is 20remains.

3. A tank with a capacity of 500 gal. originally contains 200 gal. of water with 100 lb. of salt in solution. Water containing 1 lb. of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flow out of the tank at a rate of 2 gal/min. Find the amount of salt in the tank at any time prior to the instant when the solution begins to overflow. Find the concentration (in pounds per gallong) of salt in the tank when it is on the point of overflowing. Compare this concentration with the theoretical limiting concentration if the tank had infinite capacity.