<u>Solutions</u> to Mathematics 166, Exam 1
Spring Semester, 1992
February 17, 1992

1. Evaluate each of the following integrals.

a) $\int xe^{x^2}dx = 12e^{x^2} + C$

b) $\int \sin 2x 10 + \cos 2x dx = -12 \int d(10 + \cos 2x) 10 + \cos 2x = -12 ln(10 + \cos 2x) + C$

c) $\int xx - 1dx = \int (1 + 1x - 1)dx = x + ln|x - 1| + C$

d) $\int \sin(\cos 2x) \sin 2x dx = -12 \int \sin(\cos 2x) d(\cos 2x) = 12 \cos(cos 2x) + C$

e) $\int_0^{14} 1\sqrt{1 - 4x^2}dx = 12 \int_0^{14} d(2x)\sqrt{1 - (2x)^2} = 12 \sin^{-1}(2x)|_0^{14} = 12(\pi 6 - 0) = \pi 12$

f) $\int 3x^2 + 6x^3 + 6x + 5dx = \int d(x^3 + 6x + 5)x^3 + 6x + 5 = ln|x^3 + 6x + 5| + C$

g) $\int (\sqrt{x} + 2\sqrt{x})dx = \int x^{12}dx + 2\int x^{-12}dx = x^{12} + 112 + 1 + 2x^{-12} + 1 - 12 + 1 + C = 23x^{32} + 4x^{12} + C$

h) $\int_0^{\pi^2} \sin\sqrt{x}\sqrt{x}dx = 2\int \sin\sqrt{x} \ \ d(\sqrt{x}) = -2(\cos\sqrt{x})|_0^{\pi^2} = -2(-1 - 1) = 4$

2. We have that

$I = \int 2x + 2(x^2 + 2x + 5)^2 dx + \int 1(x^2 + 2x + 5)^2 dx$

$= (x^2 + 2x + 5)^{-2+1} - 2 + 1 + \int 1[(x + 1)^2 + 4]^2 dx$

$= -1x^2 + 2x + 5 + \int 14^2[(x + 12)^2 + 1]^2 dx$

(Let $x + 12 = t$.)

$= -1x^2 + 2x + 5 + 24^2 \int 1(t^2 + 1)^2 dt$

$= -1x^2 + 2x + 5 + 18[12tt^2 + 1 + 12\int 1t^2 + 1dt]$

$= -1x^2 + 2x + 5 + 116x + 12(x + 12)^2 + 1 + 116\tan^{-1}(x + 12) + C.$

3. We have:

$$1x^2(x^2 + 1) = Ax + Bx^2 + Cx + Dx^2 + 1$$

Therefore $1 = Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2$ or $1 = (A+C)x^3 + (B+D)x^2 + Ax + B$ or $B = 1, \ \ A = 0, \ \ C = 0, \ \ D = -1$

Thus $I = \lim_{b\to\infty}\left[\int_1^b dxx^2 - \int_1^b dxx^2+1\right] = \lim_{b\to\infty}\left[-x^{-1} - tan^{-1}x\right]_1^b = \lim_{b\to\infty}\left[-b^{-1}+1-tan^{-1}b+tan^{-1}\right.$ $1 - \pi 2 + \pi 4 = 1 - \pi 4.$

4. If we multiply by e^{-x^4} we obtain $\left(ye^{-x^4}\right)' = x^3e^{-x^4}$ Thus, $ye^{-x^4} = \int x^3e^{-x^4}dx + C =$ $y(x) = 14 + ce^{x^4}$. Since $-1 = y(0) = -14 + C$, we have $C = -34$

and $y(x) = -14(1+3e^{x^4})$.

5. We have $\int x^ne^{-x}dx = \int x^nd(-e^{-x})dx = -x^ne^{-x} + \int e^{-x}nx^{n-1}$

$= -x^ne^{-x} + n\int x^{n-1}e^{-x}dx.$

Next $\int_0^\infty x^ne^{-x}dx = -x^ne^{-x}|_0^\infty + n\int_0^\infty x^{n-1}e^{-x}dx$

Since $\lim_{x\to\infty} x^ne^{-x} = 0$ we have $\int_0^\infty x^ne^{-x}dx = n\int_0^\infty x^{n-1}e^{-x}dx.$

By repeating the above argument, we obtain $\int_0^\infty x^ne^{-x}dx = n(n-1)(n-2)\cdots 1\cdot\int_0^\infty e^{-x}dx =$ $n!$

since $\int_0^\infty e^{-x}dx = 1.$