amsppt

Solutions to Math 126, Test 2

1a. We have

$$f(x+h)-(f(x)h=\sqrt{(x+h)^2+1}-\sqrt{x^2+1}h=(x+h)^2+1-x^2-1h(\sqrt{(x+h)^2+1}+\sqrt{x^2+1})=$$

$$2xh+h^2h(\sqrt{(x+h)^2+1}+\sqrt{x^2+1}=2x+h\sqrt{(x+h)^2+1}+\sqrt{x^2+1}h\to 0\to\longrightarrow 2x2\sqrt{x^2+1}$$

Therefore $f'(x)=x\sqrt{x^2+1}$.

1b. We have that

$$f(x)=\{\ sinx, x\in[2k\Pi,(2k+1)\Pi]$$

At the points $x=k\pi, k\in Z, f(x)$ is not differentiable since the derivative from the right is equal to 1 and the derivative from the left is -1. If $x\neq k\pi$, then

$$f'(x)=\{\ cosx, x\in(2k\pi,(2k+1)\pi)$$

By applying L'Hopital's rule three times we have $\lim_{x\to 0}(1x\sin x-1x^2)=\lim_{x\to 0}1-\cos x2x\sin x+$

$$\lim_{x\to 0}\sin x2\sin x+4x\cos x-x^2\sin x=\lim_{x\to 0}\cos x6\cos x-6x\sin x-x^2\cos x=16.$$

2b. We have that $W(x)\cong W(0)+W'(0)x$. Since

$$W'(x)=-2mgR^2(R+x)^3$$

We have $W(x)\cong mg-2mgRx=mg(1-2xR)$.

3a. We have $f'(x)=20x^4-20x^3=20x^3(x-1)=0\ if\ x=0,1$. $f''(x)=80x^3-60x^2=20x^2(4x-3)=0\ if\ x=0,3$. Thus, $f'(x)<0$ for $x\in(0,1)$ and $f'(x)>0$ for $x\in(-\infty,0)\cup(1,\infty)$. $f''(x)>0\ for\ x>34$. $f''(x)<0\ for\ x<34, x\neq 0$. $f(0)=2\ and\ f(1)=1$.

3b. Since f is decreasing on $(0,1)$, increasing on $(1,\infty)$ and $f(1)=1>0$ we have that $f(x)\geq 1>0$ on $(0,\infty)$. Thus, there is no real zero of $f(x)$ in $0,\infty)$. Since $f(0)=2$ and $f(-1)=-7$ by the intermediate value theorem f has a zero in the interval $(-1,0)$. Now we will show that there is no other real zero of $f(x)$ in $(-\infty,0)$. In fact, if there were two zeros $x_1,x_2\in(-\infty,0), x_1<x_2$ then, by the mean value theorem, we would have $0=f(x_2)-f(x_1)=f'(\xi)(x_2=x_1)$ for some $\xi\in(x_1,x_2)$.

This would imply $f'(\xi)=0,\xi<0$ which is impossible since f' vanishes only for $x=0\ or\ 1$.

4. Let r be the radius of the base and h the height of the cylinder. We have Volume $V=\pi r^2h$. Surface area $S=2\pi rh+2\pi r^2$. If V is fixed, then $h=V\pi r^2$ and $S=S(r)=2\pi rV\pi r^2+2\pi r^2=2VV+2\pi r^2$.

Want to minimize $S(r)$ for $r \in (0, \infty)$. We have

$$S'(r) = -2Vr^2 + 4\pi r \; and \; S'(r) = 0 \; if \; r = \sqrt[3]{V2\pi}.$$

Since $S'(r) > 0$ for $r > \sqrt[3]{V2\pi}$ and $S'(r) < 0$ for $r < \sqrt[3]{V2\pi}$ we have that $S(r)$ is minimum for $r = \sqrt[3]{V2\pi}$.

Thus the surface area takes its minimum for $r = \sqrt[3]{V2\pi}$ and $h = V(V2\pi)^2 3$.

5a. Let $f(x) = 11 + x^2$. We have $f'(x) = -2x(1 + x^2)^2 = 0 \quad$ Iff $x = 0$.

Since $f'(x) > 0$ for $x < 0$ and $f'(x) < 0$ for $x > 0$, f takes its global maximum at $x = 0$. Thus $SupA = f(0) = 1$. Since $f(x) > 0$ for all $x \in R$ and $\lim_{x \to \pm\infty} f(x) = 0$ we have $infA = 0$.

5b. By the chain rule, we have $f'(x) = \cos(g(x)) \cdot g'(x)$. Thus, $f'(1) = \cos(g(1))\, g'(1) = \cos(0) \cdot g'(1) = 1 \cdot 5 = 5$.