Math 166: Honors Calculus II Name:________________________

Exam I *Mar. 2, 1995*

There are 6 questions with a total of 25 parts. Each part is worth 5 points for a total of 125 points.

1. Define each of the following functions precisely, including their domains.

   a) $\log(x)$

   b) The natural exponential function.

   c) a^x for $a > 0$.

   d) $\sinh(x)$

   e) $\arctan(x)$

   f) $T_n f(x; a)$

2. State each of the following theorems precisely.

   a) The First Fundamental Theorem of Calculus.

   b) The Theorem on Derivatives of Inverse Functions.

   c) Taylor's Theorem with Integral Remainder.

3. Compute the following derivatives

a) $\dfrac{d}{dx}\left(\int_{x^4}^{4^x} \sqrt{1+t^4}\, dt\right).$

b) $\dfrac{d}{dx}\left(\sqrt{x}^{\sqrt{x}}\right).$

c) $\dfrac{d}{dx}\operatorname{arcsec}(x)$ (Hint: use 2b)

4. Compute the following integrals.

a) $\displaystyle\int \frac{\sin x}{3+\cos x}\,dx$

b) $\displaystyle\int \cos^4(x)\,dx$

c) $\displaystyle\int x^2 e^x\,dx$

d) $\int \log x \, dx$

e) $\int \frac{1}{\sqrt{x - x^2}} \, dx$

f) $\int \frac{1}{(x-2)(x-3)} \, dx$

5. a) Give the partial fraction decomposition of $\dfrac{3x^3 + 4x^2 + 2x + 1}{x^2(x^2 + 1)}$.

b) Convert to an integral involving trignometric functions (do not integrate): $\displaystyle\int \frac{x}{\sqrt{x^2 - 4x + 20}}\, dx$.

c) Convert to an integral involving rational functions (do not integrate): $\displaystyle\int \frac{\sin x}{2 + \sin x}\, dx$.

6. In a)–c) find the Taylor polynomials.

a) $T_{n+1}\left(\dfrac{x^2}{(1-x)^2}\right)$ (Hint: $\dfrac{d}{dx}\dfrac{1}{1-x} = \dfrac{1}{(1-x)^2}$).

b) $T_n\left(\log\dfrac{(1-x)^3}{(1+x)^5}\right)$

c) $T_4(e^x \cos x)$

d) Let $f(x) = \cos(x)$. Find n such that $|E_n f(x; 0)| \leq 10^{-5}$ for all $x \in [0, \pi/2]$.