Math 166: Honors Calculus II Name:________________________
Exam II *Apr. 20, 1995*

There are seven problems with a total of 24 parts. Each part is worth 5 points for a maximum of 120 pts.

1. (20 pts) Define the following

   a) $\lim_{x \to a} f(x) = \infty$

   b) $\lim_{n \to \infty} a_n = L$

   c) $f(x) = o(g(x))$ as $x \to a$.

   d) $\sum_{n=1}^{\infty} a_n = S$.

2. (25 pts) State the following theorems precisely.

a) The Monotone Convergence Theorem.

b) The Simple Divergence Test.

c) The Limit Comparison Test.

d) The Integral Test.

e) The Ratio Test.

3. (10 pts) State and prove L'Hopital's Theorem.

4. (5 pts) Simplify $\dfrac{1+2x-3x^2+o(x^3)}{1-4x+5x^2+o(x^3)}$ into the form $a_0 + a_1 x + o(x)$.

5. (25 pts) Compute the following limits.

a) $\lim_{x \to 0} \frac{e^{g(x)} - e}{g(x) - 1}$ where $g(x)$ is a non-constant differentiable function in a neighborhood of 0 and $g(0) = 1$.

b) $\lim_{x \to 0} (1 - 2x)^{3/x}$

c) $\lim_{x \to 0} \frac{\sin(x^2) - x^2 e^{x^4}}{x^6}$

d) $\lim_{x \to \infty} \frac{x^3 - 2x + 1}{\sqrt{4x^6 + x^3 - 1}}$

e) $\lim_{n \to \infty} \frac{n + \cos(n\pi)}{n + 1}$

6. (10 pts) Compute the sum of the following series.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + (-1)^n}{\pi^{2n}}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{(n^2+1)(n^2+2n+2)}$ (telescoping sum)

7. (25 pts) Test the following series for convergence. Briefly justify your answers.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^{1/n}}$

b) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n!}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2}$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(2n-1)(2n-3)}$

e) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\log(n)}{n}$