Math 166: Honors Calculus II Name:__________________________
Final Exam *May 9, 1995*

There are 13 problems worth a total of 180 pts.

1. (10 pts) State and prove the First Fundamental Theorem of Calculus.

2. (10 pts)

   a) Compute $\int_{-2}^{0} \frac{x}{\sqrt{x+4}}\,dx$

   b) Integrate $\int e^{-x}\sin(x)\,dx$

3. (15 pts) Give complete definitions of the following functions, including domains and ranges.

   a) $\log(x)$

   b) $\log_a(x)$ for $a > 0$

c) $\exp(x)$

d) a^x for $a > 0$

e) $\tanh(x)$

f) $\arcsin(x)$

4. (10 pts)

a) Derive $\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$.

b) Let $f(x) = \frac{e^x + 1}{e^x - 1}$. Determine the interval(s) on which $f(x)$ has an inverse and derive a formula for $f^{-1}(x)$.

5. (10 pts) Compute the following derivatives.

a) $\displaystyle \frac{d}{dx} \int_{\sin(x)}^{\cos(x)} \frac{1}{\sqrt{e^x + x}}\, dx.$

b) $\displaystyle \frac{d}{dx} \log(x)^x$

6. (15 pts)

a) Compute the partial fraction decomposition of $\dfrac{3x^3 - x^2 - 1}{(x^3 + x^2 + x)(2x - 1)}$.

b) Use an appropriate substitution to transform the integral

$$\int \frac{1}{2 + \sin(\theta)}\, d\theta$$

into an integral of a rational function (do not evaluate the integral).

c) Integrate $\displaystyle\int \frac{1}{\sqrt{2x^2 + 4x + 1}}\, dx$.

7. (15 pts)

a) State Taylor's Formula with Lagrange Remainder.

b) Find $T_{2n+1}\left(\dfrac{x}{x^2+2}\right)$.

c) Find $T_6(\log(\cos(x)))$.

8. (20 pts) Compute the following limits.

a) $\lim\limits_{x\to 0} \dfrac{\sin(x^4)}{x^2(e^x-1)^2}$

b) $\lim\limits_{x\to 0} \dfrac{2^x-3^x}{x}$

c) $\lim\limits_{x\to 0} \dfrac{1}{x} - \dfrac{1}{e^x-1}$

d) $\lim\limits_{x\to\infty} \sqrt{x}\sin(\dfrac{1}{x})$

e) $\lim\limits_{n\to\infty} \dfrac{\sqrt[n]{n^2+(-1)^n}}{\sqrt{n^3+n}}$

9. (10 pts) State and prove the Monotone Convergence Theorem.

10. (15 pts) Compute the value of the following infinite series.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+2}}$

b) $\displaystyle\sum_{n=17} \frac{2+(-1)^n}{5^n}$

c) $\displaystyle\sum_{n=1}^{\infty} 2nx^{2n+1}$ for $|x| < 1$.

11. (20 pts) Test whether the following series converge absolutely, conditionally, or diverge. Justify your answers.

a) $\sum \frac{(-1)^n}{n \log(n)^s}$ for $s > 1$.

b) $\sum_{n=1}^{\infty} (-1)^n \left(1 + \frac{1}{n}\right)^{1/n}$

c) $\sum_{n=1}^{\infty} (-1)^n \frac{\log(n+1)}{\sqrt{n(n+1)}}$

d) $\sum_{n=1}^{\infty} \frac{(-2)^n n!}{n^n}$

12. (10 pts) Determine the interval of convergence for the following power series.

a) $\sum_{n=0}^{\infty}(1+(-3)^n)x^n$

b) $\sum_{n=0}^{\infty} \frac{(x+1)^n}{(n+2)3^n}$

13. (20 pts) Compute the following integrals. Justify your answers.

a) $\displaystyle\int_0^\infty xe^{-x}\,dx$

b) $\displaystyle\int_{0^+}^1 \log(x)^2\,dx$

c) $\displaystyle\int_0^2 \frac{1}{\sqrt[3]{(x-1)^2}}\,dx$

d) $\displaystyle\int_{0^+}^{1^-} \frac{\log(1-x)}{x}\,dx$ (Test for convergence only.)