Math 166: Honors Calculus II Name:______________________________

Exam II *Apr. 18, 1996*

There are seven problems with a total of 24 parts. Each part is worth 5 points for a maximum of 120 pts.

1. (20 pts) Define the following

   a) $\lim_{x \to a} f(x) = \infty$

   b) $\lim_{n \to \infty} a_n = L$

   c) $f(x) = o(g(x))$ as $x \to a$.

   d) $\sum_{n=1}^{\infty} a_n = S$.

2. (20 pts) State the following theorems precisely.

   a) The Monotone Convergence Theorem.

   b) The Theorem on Linearity of Series.

   c) The Limit Comparison Test.

   d) The Integral Test.

3. (15 pts) Prove the following statements and solve for the constants.

a) Prove that $f(x)o(g(x)) = o(f(x)g(x))$.

b) $$\frac{1+2x-3x^2+o(x^3)}{2-4x^2+5x^3+o(x^3)} = a_0 + a_1x + a_2x^2 + o(x^2) \text{ as } x \to 0.$$

c) $$\frac{x^3e^{x^6} - \sin(x^3)}{x^9} = a_0 + o(1) \text{ as } x \to 0.$$

4. (10 pts) State and prove L'Hôpital's Theorem.

5. (20 pts) Compute the following limits. Be sure to justify each step.

a) $\lim_{x \to 0} (1 + h(x))^{1/h(x)}$ where $h(x)$ is a non-constant continuous function in a neighborhood of 0 and $h(0) = 0$.

b) $\lim_{x \to 0} \dfrac{x - \tan(x)}{x - \sin(x)}$

c) $\lim_{x \to \infty} \log(1 + 3x^2) - \log(1 + \sqrt{1 + 4x^4})$

d) $\lim_{n \to \infty} \dfrac{n + (-1)^n}{n + 1} \qquad (n \in)$

6. (15 pts) Compute the sum of the following series.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + (-1)^n}{3^{2n}}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+3}{(n+1)^2(n+2)^2}$ (telescoping sum)

c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n-1}}{2n-1}$ for any $|x| < 1$. (You may use integration or differentiation term by term.)

7. (20 pts) Test the following series for convergence. Justify your answers.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/n}}$

b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^3}{n!}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2+1}}{n^2}$

d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\log(n)^s}$